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既定限量退回與備援政策下具隨機性需求之易腐性商品的最佳訂購模型

OPTIMAL ORDERING MODEL FOR PERISHABLE ITEMS WITH STOCHASTIC DEMAND GIVEN A LIMITED RETURNS AND BACKUP POLICY

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摘要

由於高競爭性之易腐性產品經常面對相當程度起伏波動的市場需求，使得零售商的獲利性往往受到不利衝擊。鑑於此一情況，供應商目前已建立多種主要基於實現通路合作與風險分擔之政策，以之做為有效工具來減輕需求不確定風險，如此將極可能有助於增進整體通路利益。其中一種政策為供應商承諾零售商許可限量退回及提供限量備援存貨，藉以激勵零售商下訂更多訂單作為回報。此一互利合作政策對於未售出單元之殘值通常所剩無多的易腐性商品而言尤其具備價值性與必要性。為了達成此一目的，本研究延伸傳統之報童模型（Newsvendor model）以併入該一具對數常態隨機性需求之易腐性商品的限量退回與備援政策。經過周詳推導之後，本研究最終發展出一有效與實用的訂購模型，可用以最佳化訂購數量與最大化零售商的預期利潤，透過數值實例，本研究實證展示所發展之最佳訂購模型的可行性與效益性。

關鍵字：存貨管制、報童模型、退回與備援政策、易腐性商品、對數常態隨機性需求

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ABSTRACT

Competitive perishable items frequently face such sizably volatile market demand that retailer profitability is frequently unfavorably impacted. Accordingly, suppliers have presently created various policies, which are primarily based on carrying out channel coordination and risk sharing, to serve as effective instruments to ease the risk of demand uncertainty and thus potentially help improve overall channel benefits. One such policy is for suppliers to make commitments to retailers to permit limited returns and provide a limited backup inventory to incentivize retailers to place more orders in return. This coordinated policy is especially valuable and necessary for perishable items because the salvage value remaining on unsold units is often trivial. To this end, this study extends a classical newsvendor model to incorporate the limited returns and backup policy for a given perishable item with lognormal stochastic demand. Following a comprehensive deduction, an effective and practical ordering model is finally developed to optimize order quantity and maximize retailer expected profits. By means of a numerical example, this study demonstrates the workability and effectiveness of the developed optimal ordering model.

Keywords: Inventory Control, Newsvendor Model, Returns and Backup Policy, Perishable Items, Lognormal Stochastic Demand

1. Introduction

This study considers a supply chain that involves a supplier-retailer distribution channel for perishable items, which generally are characterized by a definite demand life, with examples including electronic components, fashion goods, foodstuffs, beverages, pharmaceuticals, chemicals, printed materials, etc. This study assumes that the retailer places a single-period order to purchase a quantity that matches the expected demand for a given perishable item during the upcoming selling season. However, when market demand for the given perishable item is variable and volatile, it is likely subject to a substantial difference between actual and expected demand, and thus to cause sizable losses for retailers. Accordingly, the concept and devices of channel coordination are launched to share business risk among enterprise partners, maintain long-term relationships and enhance overall supply chain advantages. Suppliers may also benefit from channel coordination to improve customer satisfaction, expand market share, and encourage retailers to purchase and sell items more actively owing to lower risk exposure. Members within a supply chain thus can share the risks and costs of demand uncertainty and boost profitability by way of channel coordination.

Among others, the returns and backup policy can be regarded as an effective mechanism to achieve channel coordination and risk sharing. Naturally, the limited returns and backup policy implies flexible supply arrangement. A returns policy refers to the supplier having a commitment to the retailer to buy back unsold units at the end of the selling season at a pre-agreed price. Presumably, this policy is especially valuable for perishable items given that unsold units often have trivial salvage value. On the other hand, a backup policy refers to the commitment of the supplier to retailers to readily replenish units for which a shortage exists at the end of the selling season for a certain additional charge. Generally, suppliers are only ready to promise limited returns and backup quantities owing to fear of assuming excessive risk. This study differs from previous researches by focusing on extending the classical newsvendor model to develop an optimal ordering model for retailers of a given perishable item where a limited returns and backup commitment by the supplier is considered and lognormal stochastic demand is assumed.

A pioneering study of returns policy by Pasternack (1985) asserted that manufacturers can achieve channel coordination with retailers by adjusting wholesale price and permissible returns, and thus mitigate double marginalization and increase supply chain joint profits. This conclusion was also endorsed by Padmanabhan and Png (1995). Subsequently, extensions and variations introduced exclusive returns policies to supply chain systems with a view to examining changes in supply chain profits given policy adoption. For example, Marvel and Peck (1995)；Emmons and Gilbert (1998)；Lee (2001), and others examined returns policies using a single-period model of returns. Furthermore, Lau and Lau (1999)；Webster and Weng (2000) attempted to modify the model to include different risk preferences and tolerances with regard to unexpected changes in market demand among channel members.

Padmanabhan and Png (1997) looked into returns policy in a non-newsvendor framework and demonstrated that it favors manufacturers by simulating retail competition. Mantrala and Raman (1999) employed the traditional “newsvendor problem” modeling framework to investigate returns policy by considering a situation where a supplier faces a retailer with multiple store outlets. Donohue (2000) discussed returns policy where a manufacturer can deliver a second production batch following an updated sales forecast. Additionally, Tsay (2001) compared returns policy against a markdown allowance policy. Taylor (2001) analyzed channel coordination in a two-period and declining price context and for three channel policies, including wholesale price protection, midlife returns and end-of-life returns, to determine how these policies are coupled together so as to optimize channel coordination. Hahn, Hwang, and Shinn (2004) dealt with retailer operating policies for a perishable product in a situation in which a retailer agrees with their supplier not to return unsold product provided the supplier offers a discount on the wholesale price.

Chen and Bell (2011) examined the potential impact of returns on these two decisions including ordering quantity of retailer and wholesale price of manufacturer, and expected profits of both manufacturer and retailer for a single-period product with stochastic demand. Lee and Rhee (2007) examined returns policy in a newsvendor framework and assumed both supplier and retailer to have limited and stochastic salvage capacities. Pasternack (2008) demonstrated that a pricing and returns policy in which a manufacturer offers retailers a partial credit for all unsold items can achieve channel coordination in a multi-retailer environment. Li, Xu, and Li (2013) devised a number of study models to look at the impact of return policy, product quality and pricing strategy of an online distributor on these two decisions including customer purchase and return. Gümüs, Ray, Yin, and Yin (2013) investigated the role played by consumer valuation of used durable products in shaping the incentive of a manufacturer to proffer retailer a returns option acting as the channel equilibrium strategy when used goods might be devaluated owing to physical deterioration (or obsolescence). Hu, Li, and Govindan (2014) considered a consignment contract that allows consumers to return non-defective products and compared the difference between vendor and retailer managed consignment inventory with and without return policy. Parvini, Atashi, Husseini, and Esfahanipour (2014) studied the effects of returns policy for a reusable product on inventory policy of all parties involved in a supply chain by incorporating both manufacturing and remanufacturing processes into the extended inventory model.

Various studies have also examined backup policies. Eppen and Iyer (1997) developed the concept of backup agreement contract, which involves two main parameters: reservation rate and penalty cost. An optimal buyer purchase policy was derived in their study by setting up a two-period dynamic programming model. Kouvelis and Li (2008) investigated the optimal replenishment cycles and the effectiveness and practicability of the backup supplier for a given product with the constant rate demand and stochastic lead time. Bassok and Anupindi (1997) analyzed a single-production periodic review inventory system with a minimum quantity commitment that optimized buyer inventory policy and purchase decision. Hou, Zeng, and Zhao (2010) (studied a buyback policy committed from backup supplier to cover situations where the supplier is subject to disruptions. Meantime, the uncertainties of demand and recurrent supply are explored to find the expected profit functions and optimal decisions of both buyer and supplier.

Besides, high market volatility in numerous industries is becoming increasingly familiar owing to intense competition, fickle consumer preferences, and rapid variations in product and processing technology. Consequently, market demand for perishable items considered here is also assumed to be subject to uncertainty and random volatility, implying it is probabilistic. Recent studies involving the newsvendor-type inventory model thus have extensively analyzed the role of probabilistic random demand. Furthermore, most probabilistic demand-related studies have adopted independent normal demand for each time period. Based on their analytical models, Bagchi and Hayya (1984)；Bagchi, Hayya, and Ord (1984)；Silver, Pycke, and Peterson (1998)；Mantrala and Raman (1999)；Tang, Rajaram, and Alptekinoglu (2004)；McCardle, Rajaram, and Tang (2004)；Chen and Chen (2009, 2010)；Jha and Shanker (2013) assumed normally distributed demand. Using a continuous demand variable for a given product following a normal distribution appears plausible since market demand is frequently aggregated from numerous individual demands (especially for perishable items) and hence reaches sufficient mass to satisfy the central limit theorem.

Nevertheless, normal distribution remains a questionable proxy for demand distribution since demand cannot be negative. Bartezzaghi, Verganti, and Zotteri (1999) asserted that, where relevant, a distribution should be sought if its field is only defined for non-negative values. The lognormal distribution can be treated as a more realistic and acceptable alternative to normal distribution because it results in a normal distribution after the logarithmic operation. A variable with a lognormal distribution can have any value between zero and infinity. This study thus assumes that market demand for perishable items following a lognormal probability distribution should be theoretically acceptable and sustainable. Notably, Benavides, Amram, and Kulatilaka (1999)；Huang, Chang, and Chou (2008) also supported the lognormal distribution, while using it to analyze demand forecasting problems in manufacturing.

As mentioned previously, this study endeavors to develop an optimal ordering model for retailers of perishable items facing volatile market demand during a single period. Meantime, the supplier adopts a limited returns and backup policy. The developed optimal ordering model can help retailers optimize order quantity of a given perishable item during the upcoming selling season to maximize expected profit. Moreover, this study conceives of a practicable returns and backup policy to realize channel coordination, which can improve retailer profitability and increase supplier market share. Importantly, because sharing of demand uncertainty and promotion of overall channel benefits are both advantageous, the limited returns and backup policy represents a useful instrument for channel coordination.

2. Model Development

2.1 Modeling Demand Forecast

As stated earlier, this study applies the Ito process to capture the demand shift via the corresponding continuous-time differential equation. The Ito process is a stochastic process that possesses the Markov property and comprises a permanent component of regular drift accounting for predictable long-term trends and a temporary component of random diffusion accounting for unpredictable stochastic volatility. Let D_t denote demand quantity during period t for a given newsvendor-style perishable item. The Ito process for stochastic variable D_t can be expressed algebraically as

$$dD_t = \mu(D_t, t)dt + \sigma(D_t, t)dz_t \qquad (1)$$

In Eq. (1), the stochastic diffusion equation $\mu(D_t, t)$ represents the regular drift component, and $\sigma(D_t, t)$ represents the random diffusion component. Furthermore, variable z_t is assumed to satisfy the standard Wiener process, and its volatility can be expressed as $dz_t = \varepsilon_t \sqrt{d\tau}$ where ε_t is a standard normal stochastic variable, and $d\tau$ represents a given small time interval.

If the diffusion component is approximately stationary (independent of time), a condition that is expected to hold for the demand process of a typical commodity over a relatively finite time horizon, it can acquire a specific form $\mu(D_t, t) = \mu D_t$ and $\sigma(D_t, t) = \sigma D_t$. The parameters μ and σ represent expected demand growth and the standard deviation of that growth, respectively, and both are constant for all time periods. Eq. (1) can then be reformulated as follows:

$dD_t = \mu D_t dt + \sigma D_t dz_t$, or

$$\frac{dD_t}{D_t} = \mu dt + \sigma dz_t . \quad (2)$$

Assume D_t is a lognormally distributed variable. Let $f = f(D_t, t) = \ln D_t$, and the following consequence can then be derived by applying Taylor expansion with respect to *f*, and Eq. (2)

$$\begin{aligned} df &= \frac{df}{dD_t}(dD_t) + \frac{1}{2}\frac{d^2 f}{dD_t^2}(dD_t)^2 + \ldots = \frac{dD_t}{D_t} - \frac{1}{2D_t^2}(dD_t)^2 + \ldots \\ &\cong \left(\mu dt + \sigma dz_t\right) - \frac{1}{2D_t^2}(dD_t)^2 \end{aligned} \quad (3)$$

By doing Euler discretization manipulation about Eq. (3); the following discrete-time model can be obtained.

$$D_t = D_0 \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma\varepsilon\sqrt{t}\right], \quad (4)$$

where

t = length of time period,

μ = expected annual demand growth rate,

σ = standard deviation of demand growth rate,

ε = a standard normal stochastic variable; that is, $\varepsilon \sim N(0,1)$.

Eq. (4) shows that the expected value and variance of $\ln D_t$ are $E[\ln D_t] = \ln D_0 + \left(\mu - \sigma^2/2\right)t$ and $Var[\ln D_t] = \sigma^2 t$, respectively.

2.2 Formulating Ordering Model

Definitions of symbols for the analytical model are given below.

T = length of selling period for a given perishable item,

p = unit selling price (retail price) for a given perishable item,

c = unit purchasing cost (wholesale price) for a given perishable item,

s = unit salvage value for a given perishable item $(s < c)$,
v = unit shortage cost for a given perishable item $(v \geq p - c)$,
D= demand quantity during the upcoming selling period for a given perishable item,
D_o= actual demand quantity during the previous selling period for a given perishable item,
M= returns limit for a given perishable item,
N= backup inventory limit for a given perishable item,
r = unit refund on returns for a given perishable item $(r \leq c)$,
b = unit premium on backup inventory for a given perishable item $(b \leq p - c)$.

In the case of returns and backup policy, the refund on returns can be considered as additional revenue to compensate for the loss of retailer when the supply exceeds the demand, while the premium on backup inventory considered as an additional cost when the backup agreement is activated. Both unit refund and premium are assumed as a known and constant parameter in the presented ordering model because they are generally determined in practice through a negotiation between supplier and retailer.

Let Q represent order quantity during the upcoming selling period given a limited returns and backup policy, respectively. Expressed formally, the profit function during the upcoming selling period for the retailer in the typical newsvendor model depends principally on demand and order quantity, and can be formulated as follows:

$$R = \begin{cases} (p-c)D-(c-r)M-(c-s)\left[Q-(D+M)\right] & if \quad 0 \leq D < Q-M \\ (p-c)D-(c-r)(Q-D) & if \quad Q-M \leq D \leq Q \\ (p-c)Q+(p-c-b)(D-Q) & if \quad Q \leq D \leq Q+N \\ (p-c)Q+(p-c-b)N-v\times\left[D-(Q+N)\right] & if \quad D > Q+N \end{cases}$$

alternatively,

$$R = \begin{cases} (p-s)D+(r-s)M-(c-s)Q & if \quad 0 \leq D \leq Q-M \\ (p-r)D-(c-r)Q & if \quad Q-M \leq D \leq Q \\ (p-c-b)D+bQ & if \quad Q \leq D \leq Q+N \\ (p-c+v)Q+(p-c-b+v)N-v\times D & if \quad D > Q+N \end{cases} \qquad (5)$$

Retailer expected profit is then derived from Eq. (5) and rendered as follows:

$$\begin{aligned}E[R]=&(p-s)\times\int_0^{Q-M}D\times f(D)dD+(r-s)M\times\int_0^{Q-M}f(D)dD-(c-s)Q\times\int_0^{Q-M}f(D)dD\\&+(p-r)\times\left(\int_0^{Q}D\times f(D)dD-\int_0^{Q-M}D\times f(D)dD\right)-(c-r)Q\times\left(\int_0^{Q}f(D)dD-\int_0^{Q-M}f(D)dD\right)\\&+(p-c-b)\times\left(\int_Q^{\infty}D\times f(D)dD-\int_{Q+N}^{\infty}D\times f(D)dD\right)+bQ\times\left(\int_Q^{\infty}f(D)dD-\int_{Q+N}^{\infty}f(D)dD\right)\\&+(p-c+v)Q\times\int_{Q+N}^{\infty}f(D)dD+(p-c-b+v)N\times\int_{Q+N}^{\infty}f(D)dD-v\times\int_{Q+N}^{\infty}D\times f(D)dD\end{aligned}.$$

Through properly merging, it turns out that

$$\begin{aligned}E[R]=&(r-s)\times\left[\int_0^{Q-M}D\times f(D)dD-(Q-M)\times\int_0^{Q-M}f(D)dD\right]\\&+(p-r)\times\int_0^{Q}D\times f(D)dD-(c-r)\times Q\times\int_0^{Q}f(D)dD\\&+(p-c-b)\times\int_Q^{\infty}D\times f(D)dD+bQ\times\int_Q^{\infty}f(D)dD\\&-(p-c-b+v)\times\left[\int_{Q+N}^{\infty}D\times f(D)dD-(Q+N)\times\int_{Q+N}^{\infty}f(D)dD\right]\end{aligned}. \quad (6)$$

As stated above, the demand during a specified selling period for a given perishable item is assumed to be lognormally distributed. The probability density function of the demand variable thus can be expressed as Eq. (7):

$$f(D)=\frac{1}{D}\frac{1}{\sigma\sqrt{T}\sqrt{2\pi}}e^{\frac{-(\ln D-E[\ln D])^2}{2\sigma^2T}}. \quad (7)$$

Eq. (6) thus can be worked out by first separately solving the eight underlying components embraced in the equation, as detailed below, followed by properly linking these components to recover the original expressions.

a. $\int_Q^{\infty}f(D)dD$ and $\int_{Q+N}^{\infty}f(D)dD$

This component can be algebraically obtained as follows:

$$\int_Q^{\infty}f(D)dD=\int_Q^{\infty}\frac{1}{D}\frac{1}{\sigma\sqrt{T}\sqrt{2\pi}}e^{\left[\frac{-(\ln D-E[\ln D])^2}{2\sigma^2T}\right]}dD. \quad (8)$$

Let $\ln D=s$, $E(\ln D)=\bar{s}$ and $\sigma\sqrt{T}=u$; Eq. (8) can then be transformed into the following expression.

$$\int_Q^\infty \frac{1}{D}\frac{1}{u\sqrt{2\pi}} e^{\frac{-(s-\bar{s})^2}{2u^2}} dD. \tag{9}$$

Furthermore, if $w=\frac{(s-\bar{s})}{u}$, then $dw=\frac{1}{uD}dD$ through differentiation. The lower bound of the integral for w is accordingly transformed as Eq. (10):

$$=\frac{\ln Q - E[\ln D]}{u}=\frac{\ln Q-\ln D_0-\mu T+\sigma^2 T/2}{\sigma\sqrt{T}}$$

$$=-\frac{\ln(D_0/Q)+\left(\mu-\sigma^2/2\right)T}{\sigma\sqrt{T}}=-d_{11}. \tag{10}$$

Again, after applying dw and the lower bound of the integral for w in Eq. (9), the expression can be reformulated as

$$\int_Q^\infty \frac{1}{D}\frac{1}{u\sqrt{2\pi}} e^{\frac{-(s-\bar{s})^2}{2u^2}} dD=\int_{-d_1}^\infty \frac{1}{\sqrt{2\pi}} e^{\frac{-w^2}{2}} dw = 1-N(-d_{11})=N(d_{11}), \tag{11}$$

where function $N(.)$ denotes the cumulative probability function for a standardized normal variable.

Likewise, the following expression can be calculated via the same procedure.

$$\int_{Q+N}^\infty f(D)dD = N(d_{21}), \tag{12}$$

$$d_{21}=\frac{\ln[D_0/(Q+N)]+\left(\mu-\sigma^2/2\right)T}{\sigma\sqrt{T}}.$$

b. $\int_Q^\infty D\times f(D)dD$ and $\int_{Q+N}^\infty D\times f(D)dD$

The component can be deduced through a similar technique. First, the component can be expanded as Eq. (13):

$$\int_Q^\infty D\times f(D)dD=\int_Q^\infty \frac{1}{\sigma\sqrt{T}\sqrt{2\pi}} e^{\frac{-(\ln D - E[\ln D])^2}{2\sigma^2 T}} dD. \tag{13}$$

The integral can be obtained via the same procedure used for component (1):

$$\int_Q^\infty \frac{1}{u\sqrt{2\pi}} e^{\frac{-(s-\bar{s})^2}{2u^2}} dD = D_0 e^{-\ln D_0} \int_Q^\infty e^{\ln D} \frac{1}{D} \frac{1}{u\sqrt{2\pi}} e^{\frac{-(s-\bar{s})^2}{2u^2}} dD$$

$$= D_0 e^{\mu T} \int_Q^\infty \frac{1}{D} \frac{1}{u\sqrt{2\pi}} e^{\frac{-\left[s-\left(\bar{s}+u^2\right)\right]^2}{2u^2}} dD \,. \qquad (14)$$

Let $y = \dfrac{s-\left(\bar{s}+u^2\right)}{u}$, then $dy = \dfrac{1}{u} ds = \dfrac{1}{u} d\left(\ln D\right) = \dfrac{1}{uD} dD$ can be obtained. Similarly, the lower bound of the integral for y is transformed as Eq. (15):

$$= \frac{s-\left(\bar{s}+u^2\right)}{u} = \frac{\ln Q - \left(E[\ln D] + \sigma^2 T\right)}{\sigma\sqrt{T}} = \frac{\ln Q - \left(\ln D_0 + \mu T - \sigma^2 T/2 + \sigma^2 T\right)}{\sigma\sqrt{T}}$$

$$= -\frac{\ln D_0 - \ln Q + \mu T + \sigma^2 T/2}{\sigma\sqrt{T}} = -\frac{\ln\left(D_0/Q\right) + \left(\mu + \sigma^2/2\right)T}{\sigma\sqrt{T}} = -d_{12} \,. \qquad (15)$$

Including dy and the lower bound of the integral for y in Eq. (14) yields the following equation:

$$\int_Q^\infty D \times f(D) dD = D_0 e^{\mu T} \times \int_{-d_{u2}}^\infty \frac{1}{\sqrt{2\pi}} e^{\left(\frac{-y^2}{2}\right)} dy = D_0 e^{\mu T} N(d_{12}) \,. \qquad (16)$$

Similarly, the following expressed can be obtained by following the deductive procedure mentioned-above.

$$\int_{Q+N}^\infty D \times f(D) dD = D_0 e^{\mu T} N(d_{22}) \,, \qquad (17)$$

$$d_{22} = \frac{\ln\left[D_0/(Q+N)\right] + \left(\mu + \sigma^2/2\right)T}{\sigma\sqrt{T}} \,.$$

c. $\int_0^Q f(D) dD$ and $\int_0^{Q-M} f(D) dD$

Because the deduction of this component closely resembles that for component (1), the specific procedure is not detailed here. The following close-form formula is also identified:

$$\int_0^Q f(D)dD = N(-d_{11}) = 1 - N(d_{11}) . \quad (18)$$

$$\int_0^{Q-M} f(D)dD = N(-d_{31}) = 1 - N(d_{31}) \quad (19)$$

$$d_{31} = \frac{\ln\left[D_0 /(Q-M)\right] + \left(\mu - \sigma^2/2\right)T}{\sigma\sqrt{T}}$$

d. $\int_0^Q D \times f(D)dD$ and $\int_0^{Q-M} D \times f(D)dD$

This component is identified as well much like component (2). Thus, only the final outcome is rendered, as follows:

$$\int_0^Q D \times f(D)dD = D_0 e^{\mu T} \times N(-d_{12}) = D_0 e^{\mu T} \times \left[1 - N(d_{12})\right]. \quad (20)$$

$$\int_0^{Q-M} D \times f(D)dD = D_0 e^{\mu T} \times N(-d_{32}) = D_0 e^{\mu T} \times \left[1 - N(d_{32})\right]. \quad (21)$$

$$d_{32} = \frac{\ln\left[D_0 /(Q-M)\right] + \left(\mu + \sigma^2/2\right)T}{\sigma\sqrt{T}}$$

By applying the results of Eqs. (11), (12), (16), (17), (18), (19), (20) and (21) in Eq. (6), the expected retailer profit can be rewritten as Eq. (22).

$$\begin{aligned} E[R] = {} & (r-s)\left[(Q-M)N(d_{31}) - D_0 e^{\mu T} N(d_{32})\right] + (c+b-r)\left[QN(d_{11}) - D_0 e^{\mu T} N(d_{12})\right] \\ & + (p-c-b+v)\left[(Q+N)N(d_{21}) - D_0 e^{\mu T} N(d_{22})\right] + (p-s)D_0 e^{\mu T} + (r-s)M - (c-s)Q \end{aligned}, \quad (22)$$

where

$$d_{11} = \frac{\ln(D_0 / Q) + \left(\mu - \sigma^2/2\right)T}{\sigma\sqrt{T}} ,$$

$$d_{12} = \frac{\ln\left(D_0 / Q\right) + \left(\mu + \sigma^2/2\right)T}{\sigma\sqrt{T}} ,$$

$$d_{21} = \frac{\ln\left[D_0/(Q+N)\right] + \left(\mu - \sigma^2/2\right)T}{\sigma\sqrt{T}},$$

$$d_{22} = \frac{\ln\left[D_0/(Q+N)\right] + \left(\mu + \sigma^2/2\right)T}{\sigma\sqrt{T}},$$

$$d_{31} = \frac{\ln\left[D_0/(Q-M)\right] + \left(\mu - \sigma^2/2\right)T}{\sigma\sqrt{T}},$$

$$d_{32} = \frac{\ln\left[D_0/(Q-M)\right] + \left(\mu + \sigma^2/2\right)T}{\sigma\sqrt{T}}.$$

Theorem. The optimal ordering model developed can work out one and only one optimal solution known as concavity.

Proof. Taking the first derivative of $E[R]$ with respect to order quantity Q reveals that

$$\begin{aligned}\frac{dE[R]}{dQ} &= (r-s)\left[N(d_{31}) - \frac{n(d_{31})}{\sigma\sqrt{T}} + D_0 e^{\mu T}\frac{n(d_{32})}{\sigma\sqrt{T}(Q-M)}\right] + (c+b-r)\left[N(d_{11}) - \frac{n(d_{31})}{\sigma\sqrt{T}} + D_0 e^{\mu T}\frac{n(d_{12})}{\sigma\sqrt{T}Q}\right] \\ &\quad + (p-c-b+v)\left[N(d_{21}) - \frac{n(d_{21})}{\sigma\sqrt{T}} + D_0 e^{\mu T}\frac{n(d_{22})}{\sigma\sqrt{T}(Q+N)}\right] - (c-s) \\ &= (r-s)N(d_{31}) + (c+b-r)N(d_{11}) + (p-c-b+v)N(d_{21}) - (c-s)\end{aligned} \quad ,（23）$$

where $n(d_{ij}) = \frac{1}{\sqrt{2\pi}} e^{-d_{ij}^2/2}$. The maximal value of $E[R^*]$ occurs at $Q*$, satisfying $dE[R]/dQ = 0$ for Eq. (23). More specifically, the optimal order quantity can be solved by means of a numerical method. Furthermore,

$$\frac{dE[R]^2}{d^2Q} = -\left[(r-s)\frac{n(d_{31})}{\sigma\sqrt{T}(Q-M)} + (c+b-r)\frac{n(d_{11})}{\sigma\sqrt{T}Q} + (p-c-b+v)\frac{n(d_{21})}{\sigma\sqrt{T}(Q+N)}\right] < 0, \quad（24）$$

the concavity can thus be verified through Eq. (24).

2.3 Estimating Model Parameters

The expected growth rate of demand μ and standard deviation of growth rate σ in Eq. (3) can be estimated using the sample estimates $\hat{\mu}$ and $\hat{\sigma}$, based on historical demand data. This study introduces a straightforward and practical estimate. Assuming a sample of demand data covering N time periods, that is, $D_1, D_2, ..., D_N$, where each period lasts $\Delta\tau$, for considered perishable item, the logarithmic growth rate of demand r_t for the demand time series during period t is as Eq. (25):

$$r_t = \ln\left(\frac{D_t}{D_{t-1}}\right); \quad t = 2,3,...,N. \tag{25}$$

Additionally, $r_t,\ t = 2,3,...,N$ all share an independent and identical normal distribution with mean $\bar{r}$ and standard deviation s. Accordingly, estimates $\hat{\mu}$ and $\hat{\sigma}$ can be calculated as Eqs. (26) and (27):

$$\hat{\mu} = \frac{\bar{r}}{\Delta\tau} + \frac{s^2}{2\Delta\tau}, \text{ and } \hat{\sigma} = \frac{s}{\sqrt{\Delta\tau}}, \tag{26}$$

$$\bar{r} = \frac{\sum_{t=2}^{N} r_t}{N-1}, \text{ and } s = \sqrt{\frac{\sum_{t=2}^{N}(r_t - \bar{r})^2}{N-2}}. \tag{27}$$

3. Numerical Experiment

This section outlines a numerical example demonstrating the proposed analytical model for optimizing order quantity to maximize retailer expected profit. The model parameters required in the numerical example are arranged for hypothetical perishable items, and a corresponding value set is designed that comprises ($D_0, \mu, \sigma, T, p, c, s, v, M, N, r, b$) = (10,000, 0.25, 0.3, 0.5, \$500, \$300, \$50, \$300, 2500, 2000, \$200, \$100). The optimal order quantity and maximal expected profit for the above parameter settings using the presented analytical method and a numerical solution procedure offered by MS-Excel are solved as 11,823 units and \$1,931,763, respectively.

To examine and verify the concavity, Table 1 lists the resulting expected profits given various assumed order quantities, ranging from 6,000 to 17,000 units in increments of 500 units, as well as the optimal order quantity found in this experiment. For visibility purposes, Fig. 1 also illustrates that expected profit varies with order quantity. The variability curve demonstrates that expected profit is clearly concave, and first increases then decreases with increases in order quantity. Meanwhile, expected profit peaks at $1,931,763, which occurs with an order size of 11,823 units, consistent with the solution above. Subsequently, expected profit increases with order quantity.

Subsequently, this study conducts sensitivity analysis of core parameters, namely demand growth rate, demand volatility, returns limit, backup inventory limit, returns refund and backup charge. The aim is to clarify variation in total expected profits with changes in these parameters.

3.1 Demand growth rate and volatility

Higher demand growth rate generally boosts expected profit. However, demand volatility, commonly measured using the standard deviation of demand growth rate and used to indicate uncertainty in future market demand, often negatively impacts expected profit. Effectively, a limited returns and backup policy is created precisely to reduce demand volatility and thus improve retailer profit. Assuming all other conditions remain unchanged, expected growth rate and volatility of market demand are respectively assigned a value ranging between -50% and 100% and changing in 20% increments, and a value ranging between 5% and 95% and changing in 15% increments, respectively, to estimate the effects of optimal order quantity and expected profit. Table 2 numerically lists the results of sensitivity analysis under various combinations of demand growth rate and volatility.

Fig. 2 shows that optimal order quantity and maximal expected profit are both closely and positively linearly related with expected demand growth rate. On the contrary, optimal order quantity and maximal expected profit clearly diverge in response to changes in demand volatility. Fig. 2 also displays that the increase in demand volatility continuously and substantially negatively influences expected profit, regardless of expected growth rate. In contrast, as shown on the curve of optimal order quantity, optimal order quantity initially increases with increasing volatility, meaning retailers should extend ordering to prevent costly losses associated with shortage units in circumstances of increased demand

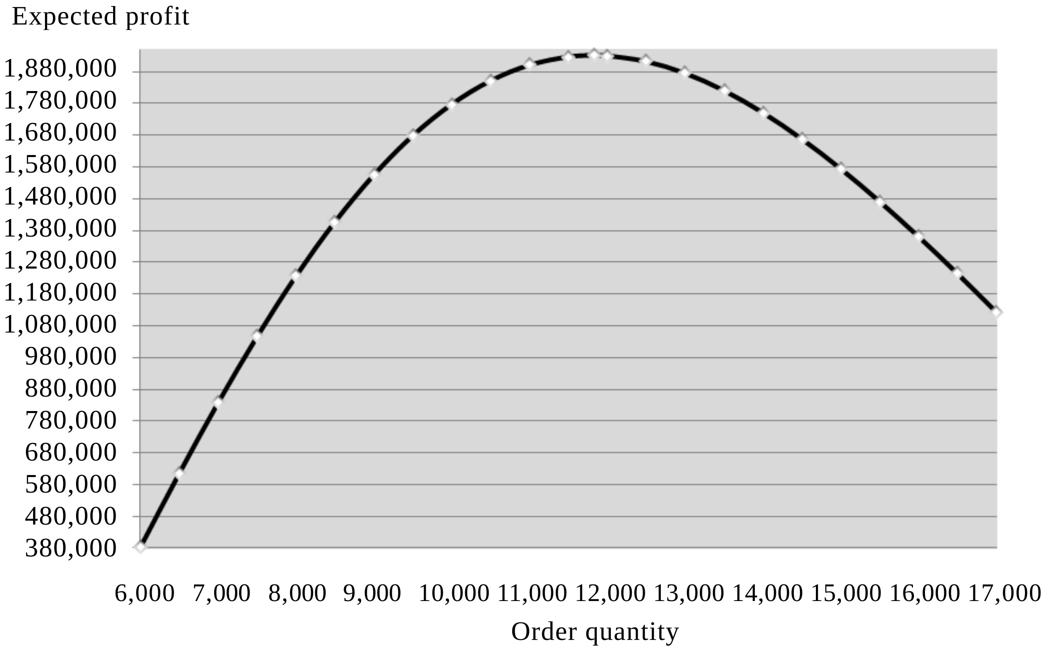

Fig. 1　Variation in expected profits over a given range of order quantities

Table 1　Comparison of expected profits for a given range of order quantities

Order quantity	Expected profit	Order quantity	Expected profit
6,000	383,462	11,823	1,931,763
6,500	616,526	12,000	1,930,416
7,000	838,821	12,500	1,912,590
7,500	1,046,906	13,000	1,875,469
8,000	1,237,485	13,500	1,821,018
8,500	1,407,705	14,000	1,751,327
9,000	1,555,369	14,500	1,668,510
9,500	1,679,020	15,000	1,574,619
10,000	1,777,912	15,500	1,471,568
10,500	1,851,914	16,000	1,361,083
11,000	1,901,405	16,500	1,244,668
11,500	1,927,184	17,000	1,123,601

Table 2 Optimal order quantities and expected profits for various combinations of expected growth rate and demand volatility

μ	σ	Q*	$E[R]$*	μ	σ	Q*	$E[R]$*
-0.50	0.05	7,783	1,535,637	0.50	0.05	12,832	2,531,836
	0.20	7,908	1,439,905		0.20	13,214	2,360,472
	0.35	8,170	1,321,601		0.35	13,464	2,043,769
	0.50	8,241	1,129,713		0.50	13,557	1,668,531
	0.65	8,208	910,972		0.65	13,505	1,272,634
	0.80	8,084	680,890		0.80	13,307	871,205
	0.95	7,877	448,048		0.95	12,967	472,913
-0.25	0.05	8,819	1,740,104	0.75	0.05	14,541	2,868,939
	0.20	9,010	1,648,290		0.20	14,989	2,655,285
	0.35	9,256	1,476,088		0.35	15,262	2,276,412
	0.50	9,325	1,245,795		0.50	15,371	1,841,245
	0.65	9,285	989,810		0.65	15,316	1,387,143
	0.80	9,145	723,899		0.80	15,093	928,937
	0.95	8,910	456,637		0.95	14,709	475,477
0.00	0.05	9,994	1,971,796	1.00	0.05	16,479	3,250,901
	0.20	10,250	1,859,910		0.20	16,997	2,984,747
	0.35	10,485	1,646,423		0.35	17,305	2,536,191
	0.50	10,558	1,373,032		0.50	17,433	2,034,055
	0.65	10,513	1,075,486		0.65	17,373	1,514,596
	0.80	10,356	769,481		0.80	17,123	992,436
	0.95	10,090	463,574		0.95	16,689	476,713
0.25	0.05	11,324	2,234,338				
	0.20	11,643	2,096,422				
	0.35	11,880	1,834,783				
	0.50	11,961	1,513,248				
	0.65	11,912	1,169,282				
	0.80	11,736	818,329				
	0.95	11,435	468,969				

volatility. Later, when volatility reaches around 0.65, optimal order quantity decreases with increasing volatility, meaning retailers should curtail ordering to avoid excessive unsold units under high volatility. To summarize, high demand volatility generally negatively impacts retailer profitability, and in this condition a limited returns and backup policy is more helpful to ease risk of demand uncertainty, and thus retailers should negotiate as actively as possible with suppliers.

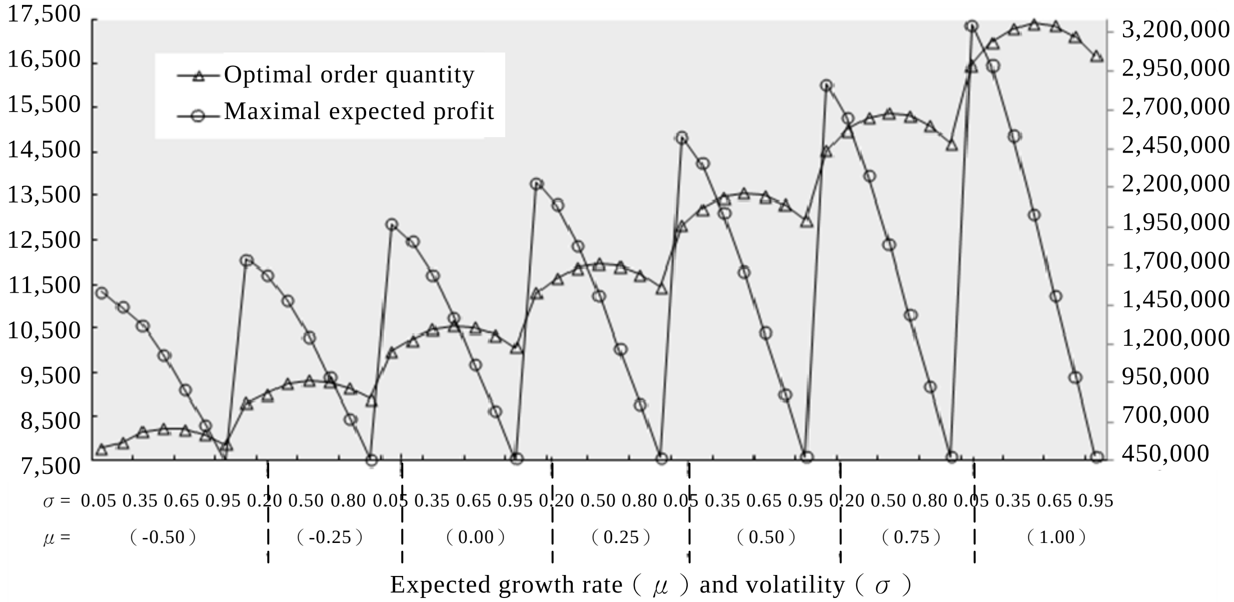

Fig. 2　Effect of changes in expected growth rate and volatility of demand on optimal order quantity and maximal expected profit

3.2 Returns and backup inventory limits

Once supplier and retailer have agreed on the returns and backup policy, returns and backup inventory limits are then determined by means of negotiation. To reduce risk, retailers should seek to maximize both quantity limits. However, on the other hand, suppliers should avoid committing themselves to accepting returns and providing backup inventory in such high volumes that too much business risk will be transferred from retailers. Therefore, supplier and retailer regularly make a deal in quantity limits by compromise. Once again, all other conditions being equal, the returns and backup inventory limits are equally assigned a value ranging between zero and 10,000 units, changing in increments of 2,000 units, to perform sensitivity analysis, and Table 3 numerically lists the results of sensitivity analysis for all designated combinations of returns and backup inventory limits. Particularly, Table 3 clearly shows that when returns and backup inventory limits are zero, meaning the returns and backup policy is unavailable, only a profit of $1,557,012 can be expected, which is considerably inferior to the maximum of $1,931,763 in the present experiment. Consequently, this study asserts that channel coordination can be achieved provided that the returns and backup policy is

Table 3 Optimal order quantities and expected profits for various combinations of returns and backup inventory limits

M	N	c	$E[R]$*	M	N	Q*	$E[R]$*
0	0	12,019	1,557,012	6,000	0	13,498	1,865,909
	2,000	11,097	1,782,423		2,000	12,439	1,977,954
	4,000	10,419	1,908,770		4,000	11,692	2,039,168
	6,000	10,004	1,965,331		6,000	11,292	2,064,913
	8,000	9,815	1,985,064		8,000	11,138	2,073,139
	10,000	9,752	1,990,611		10,000	11,093	2,075,285
2,000	0	12,527	1,736,884	8,000	0	13,601	1,869,405
	2,000	11,678	1,911,826		2,000	12,457	1,978,327
	4,000	11,075	2,002,579		4,000	11,695	2,039,214
	6,000	10,733	2,039,847		6,000	11,293	2,064,925
	8,000	10,594	2,051,816		8,000	11,138	2,073,145
	10,000	10,552	2,054,987		10,000	11,094	2,075,290
4,000	0	13,087	1,833,917	10,000	0	13,603	1,869,439
	2,000	12,202	1,967,047		2,000	12,457	1,978,327
	4,000	11,559	2,035,210		4,000	11,695	2,039,214
	6,000	11,199	2,062,872		6,000	11,293	2,064,925
	8,000	11,058	2,071,598		8,000	11,138	2,073,145
	10,000	11,016	2,073,869		10,000	11,094	2,075,290

adopted by supplier and retailer of a given perishable item, and thus substantially increase retailer profit.

Fig. 3 clearly shows that changes in returns and backup inventory limits influence optimal order quantity and maximal expected profit react differently. Whereas optimal order quantity is negatively correlated with both quantity limits, the correlation with maximal expected profit is positive. This correlation is easily explained. Presumably, smaller quantities are ordered because higher returns and backup inventory limits provide the major protection against losses arising from unsold units and shortages. Additionally, Fig. 3 also reveals that optimal order quantity and maximal expected profit remain virtually unchanged when returns and backup inventory limits exceed about 4,000 and 6,000 units, respectively. Consequently, in this experiment retailers should only need to request a returns limit of 4,000 units and backup inventory limit of 6,000 units.

3.3 Returns refund and backup premium

Returns refund and backup premium are expenses that a retailer pays to promise suppliers for the benefits of being able to return product and have access to a backup

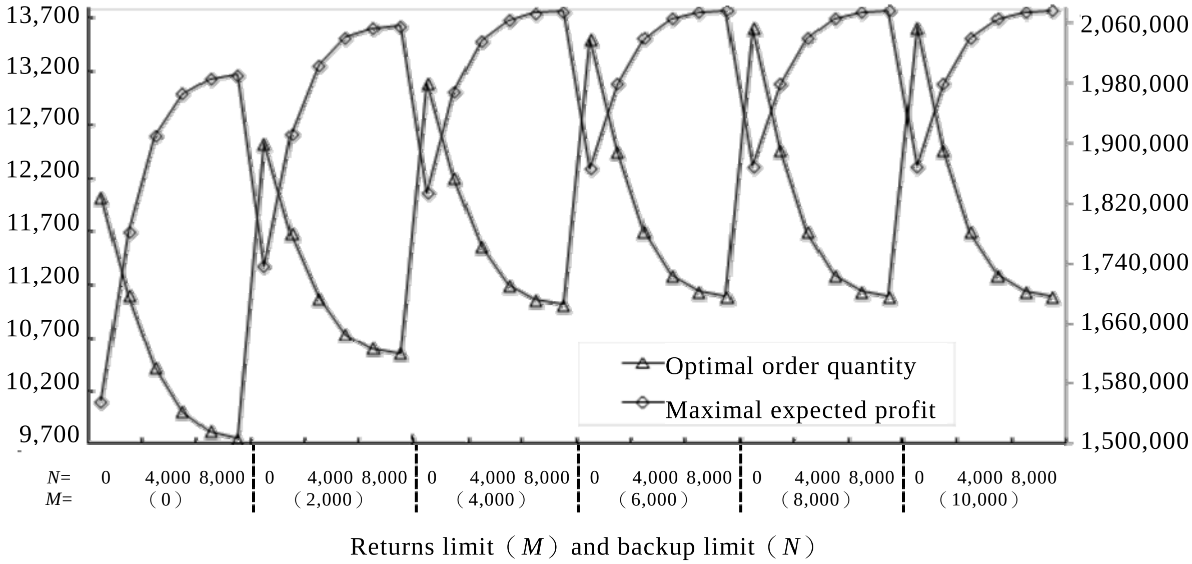

Fig. 3 Effect of changes in returns and backup inventory limits on optimal order quantity and maximal expected profit

inventory. Higher returns refund and lower backup premium are naturally more profitable for retailers. Similar to quantity limits, retailer also needs bargaining with supplier over returns refund and backup premium. Correspondingly, all other conditions being constant, returns refund and backup premium are respectively assigned values ranging between $0 and $300 and between $0 and $200, changing in increments of $100, respectively, to observe the variant effects. Table 4 numerically presents the results of sensitivity analysis for all arranged combinations of returns refund and backup charge.

Fig. 4 clearly demonstrates that both optimal order quantity and maximal expected profit increase with returns refund, consistent with expectations. Meantime, Fig. 4 also illustrates the discrepancy whereby optimal order quantity continuously grows and expected profit constantly declines when a retailer and supplier agree on higher backup premiums. In practice, refunds on returns and backup premiums are settled by hard negotiation and often depend on the comparative strength and depth of cooperation between supplier and retailer.

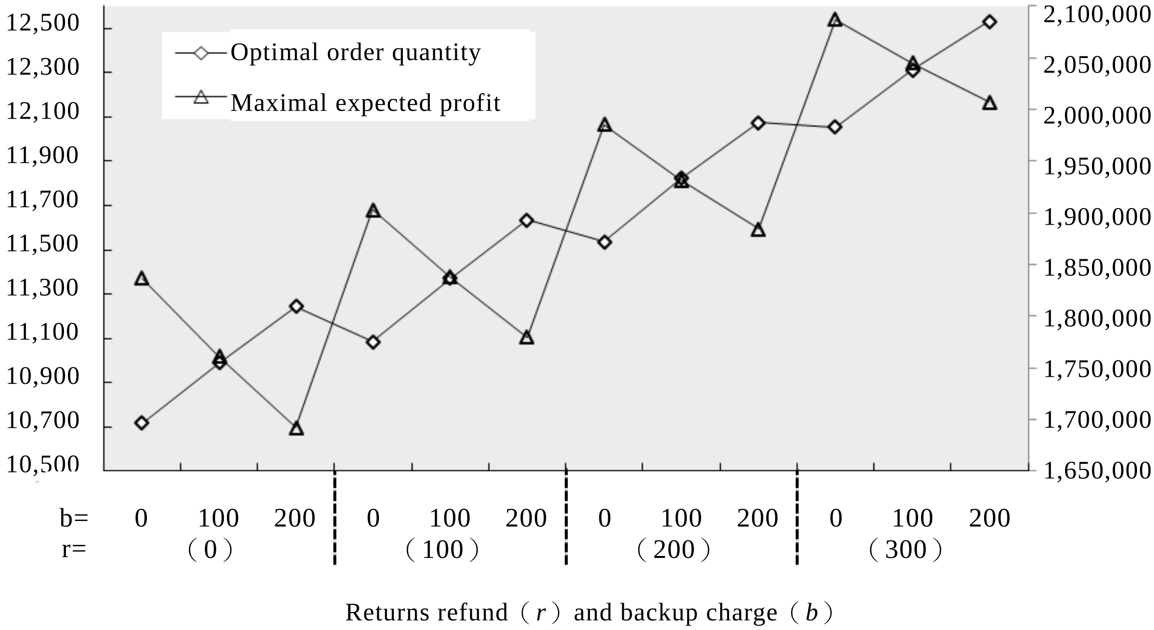

Fig. 4　Effect of changes in returns refund and backup premium on optimal order quantity and maximal expected profit

Table 4　Optimal order quantities and expected profits for various combinations of returns refund and backup premium

r	b	Q*	E[R]*	r	b	Q*	E[R]*
0	0	10,717	1,836,903	200	0	11,538	1,985,430
	100	10,989	1,760,854		100	11,823	1,931,763
	200	11,245	1,692,438		200	12,073	1,884,299
100	0	11,085	1,903,236	300	0	12,055	2,087,468
	100	11,373	1,837,925		100	12,311	2,045,030
	200	11,635	1,779,868		200	12,531	2,007,254

4. Concluding Remarks

This study extends a typical newsvendor model to incorporate the limited returns and backup policy, which has been treated as a useful instrument in fulfilling channel coordination. Consequently, as noted previously, limited returns and backup policies have been widely considered and even practiced to share risk associated with demand uncertainty and stimulate orders. Such services are especially valuable and necessary for perishable items because negligible salvage value often remained on unsold units and there existed a risk of losing customers in the event of shortages. Accordingly, this study solves the optimal order quantity for retailers of a given perishable item to maximize expected profits during the next selling period given a limited returns and backup policy. Particularly, this study assumes that market demand during a selling period follows a lognormal distribution, which is considered more feasible than a familiar normal distribution. Moreover, the Ito process is applied to model stochastic shifts in market demand.

This study develops an effective and practical analytical method for use alongside the extended newsvendor model to optimize order quantity so as to maximize expected profit during the upcoming selling period. This study applies a numerical experiment to demonstrate the workability and accuracy of the developed analytical method. Additionally, sensitivity analyses for the crucial model parameters identify some noticeable effects. To conclude, the analytical model presented in this study and the experimental findings confirm that the limited returns and backup policy assists retailers, who trade in perishable items with approximately lognormal demand, to improve their profitability. Moreover, it is especially beneficial and worthwhile for retailers of perishable items in case of high demand volatility, and additionally is associated with low salvage value and high shortage cost.

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