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存貨量相關需求及退貨政策合約下之可替代產品的供應鏈問題之研究

A NEWSVENDOR SUPPLY CHAIN FOR SUBSTITUTABLE ITEMS WITH STOCK-LEVEL DEPENDENT DEMAND AND A RETURN-POLICY CONTRACT

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摘要

本文探討分散供應鏈問題，其中某製造商供應二種可替代產品與某零售商，而此零售商在一個銷售量與存貨量相關的市場內，同時考慮產品替代性及存貨量需求刺激的效應下，銷售此二種產品。為了達到與集中供應鏈相同效果，此製造商擬定合約保

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障與此零售商，包括提供相較於分散供應鏈便宜的零售價及接受所有未售完產品的退貨政策。本文之目的乃同時藉由此二種產品訂貨量之決定、此二種產品零售價及退貨價格之協商來協調此供應鏈並創造雙贏的局面。因此，我們為此二產品分別找到零售價範圍及退貨價格範圍供雙方協商以便達到 Pareto-efficiency。

關鍵字：供應鏈、產品替代、存量貨相關之需求、退貨政策

ABSTRACT

This study addresses a decentralized supply chain whereby a manufacturer supplies two substitutable items to a retailer who sells the two items in a stock-level dependent demand market, considering the effects of product substitution and inventory demand stimulation. To operate the chain as a centralized supply chain, the manufacturer engages in a contractual commitment that not only offers the two cheaper wholesale prices but also accepts all unsold products at the end of the selling period. The objective is to coordinate the chain and create a win-win situation by means of jointly determining the two optimal order quantities, negotiating the two wholesale prices and setting the two buyback prices. To this end, a range of the two wholesale prices and a range of the two buyback prices to ensure the contract Pareto-efficiency are presented. Many managerial insights regarding the product substitution and the inventory demand stimulation are observed thereafter.

Keywords: Supply Chain, Product Substitution, Stock-Level Dependent Demand, Return Policy

1. Introduction

It is well known that if a manufacturer and a retailer, two independent entities in a supply chain, are each seeking to optimize their own profits, then the action will generate a so-called “double-marginalization” phenomenon (Spengler, 1950). This, in turn, will lead the chain to poor channel profit performances as a result of a less optimal order quantity compared to a coordinated supply chain. Therefore, contractual terms enhancing a chain’s profit efficiency have become an imperative when facing an inventory management in a supply chain setting. Two purposes of such contracts include the supply chain coordination and the Pareto-efficiency. A contract is said to coordinate a chain if the chain’s profit is maximized; a contract is said to be Pareto-efficient if each member’s profit in the chain is no worse off when the contract is in place than it would in the event of other default contracts (Bose & Anand, 2007).

A traditional price-only contract is broadly considered as a basic, simple trade-off in the marketing literature when a manufacturer does not provide any incentive to retailers, and his downstream partner then will take all responsibilities for leftover inventory at the end of the selling season. However, many researchers, such as Lariviere and Porteus (2001)；Cachon (2003)；Bernstein and Federgruen (2005), have found that the price-only contract cannot coordinate a supply chain. In contrast, a return-policy contract, which mainly mitigates the risk of over-stocking because of the market’s demand uncertainty, is a commitment made by manufacturers to accept their downstream partners’ unsold products at the end of the selling season (Padmanabhan & Png, 1995). Furthermore, Pasternack (1985), the first to analyze the manufacturer-retailer channel coordination via a return policy for a seasonal item, contended that return policies could be used as an instrument for supply chain coordination. Thus, plenty of extant publications have discussed the related policies in a supply chain setting. Emmons and Gilbert (1998) investigated the role of return policies in pricing and inventory decisions for catalogue goods. Lau, Lau, and Willett (2000) studied with the problem of demand uncertainty and return policy for a seasonal product. Tsay (1999) researched a quantity flexibility contract in a newsvendor supply chain. Yao, Wu, and Lai (2005) addressed with demand uncertainty and manufacturer return policies for style-good retailing competition. Bose and Anand (2007) contributed to a practical finding on return policies with exogenous pricing. Yao, Leung, and Lai (2008) analyzed an impact of price-sensitivity factors on a return policy when coordinating a supply chain, and Chen (2011) discussed return policies with a wholesale-price-discount contract within the context of a newsvendor problem.

Empirically, inventory demand stimulation is one of several factors that affect inventory decisions. For some items, such as toys, books, magazines, fashion apparel or 3C products, displaying a large quantity on shelf space can actually boost sales, a phenomenon first explained by Balakrishnan, Pangburn, and Stavrulaki (2008) in terms of the effects of increasing product visibility, kindling latent demand, signaling a popular product or providing an assurance of future availability. Prior to Balakrishnan et al. (2008)；Dana and Petruzzi (2001) announced that a large quantity can increase sales as consumers’ utilities increase as the item fill rate increases. A number of studies then mushroomed in connection with inventory-dependent demand when dealing with inventory decisions. Corstjens and Doyle (1981)；Bultez and Naert (1988)；Eliashberg and Steinberg (1993) addressed inventory demand stimulation considering shelf space allocation mathematical models. Gupta and Vrat (1986)；Baker and Urban (1988)；Goh (1992)；Urban (1995)；Balakrishnan, Pangburn, and Stavrulaki (2004) stressed on optimal inventory policy with stock-level dependent demand functions. Balakrishnan et al. (2008) then generalized the demand stimulation to a stochastic demand-modeling framework, capturing the influence of inventory on demand distribution. Rudi, Kapur, and Pyke (2001) developed a two-location inventory model with transshipment and local decision making. Zhao, Deshpande, and Ryan (2006) discussed emergency transshipment in decentralized dealer networks, exploring when to send and accept transshipment request. Sosic (2006) dealt the transshipment of inventories among retailers: myopic vs. farsighted stability. Stavrulaki (2011), from a retailer’s perspective, studied inventory decision in the framework of a single-period, stock-level dependent demand setting that is solved with a heuristic solution approach.

Meanwhile, many customers who visit a store with the intention of purchasing a certain item may switch to a homogeneous item if the original is out of stock. Accordingly, an inventory of an item can satisfy not only its own demand but also the demand for other items with similar features. Pasternack and Drezner (1991) investigated optimal inventory policies for substitutable commodities with stochastic demand, while Rajaram and Tang (2001) handled the impact of product substitution on retail merchandising. Netessine and Rudi (2003) proposed centralized and competitive inventory models with product substitution. Tang and Yin (2007) jointly determined ordering and pricing strategies for managing substitutable products. More recently, Stavrulaki (2011) discussed a retailer’s inventory for two substitutable products with stock-level dependent demand. However, none of the aforementioned articles explored a decentralized supply chain setting that simultaneously takes into account the effects of stock-level dependent demand, product substitution and a return-policy contract as well. Thus, this paper attempts to unearth a decentralized supply chain setting where a manufacturer supplies two substitutable items to a retailer in a stock-level dependent demand market. The two substitutable items are regarded as homogeneous such that one could be replaced with the other if one is out of stock. The stock-level dependent demand is interpreted as that more inventories will lead to more demand. At the same time, to operate the chain as a centralized supply chain, the manufacturer makes a contractual commitment that not only offers the two items at cheaper wholesale prices compared to the price-only contract, he also accepts all unsold products at the end of the selling period. The objective of this study includes how to determine the two optimal order quantities, how to negotiate the two wholesale prices and how to set the two buyback prices such that the contract can coordinate the chain as well as ensure a win-win situation for both members.

The remainder of this study is organized as follows. Assumptions and notation are given in Section 2 where relevant models responding to a price-only contract, a centralized supply chain setting and a decentralized supply chain setting with cheaper wholesale prices and buyback commitment are proposed. Theoretical analysis and developed solution methods are presented in Section 3, including negotiations for the two wholesale prices and the setting for the two buyback prices. Numerical examples are discussed in Section 4, along with many significant managerial insights. Finally, a summary, contributions of this study and potential directions for further explorations are presented in Section 5.

2. The models

The problem presented in this study is as follows. A manufacturer supplies two substitutable items to a retailer who is the follower selling the two items in a stochastic demand market. The demand for item i, $i=1,2$, is assumed to be a stock-level dependent random variable X_i in a positive range of $[a_i(Q_i), b_i(Q_i)]$ where Q_i is the order quantity and both $a_i(Q_i)$, $b_i(Q_i)$ are increasing in Q_i for item i, $i=1,2$, such that $0 < a_i(Q_i) < b_i(Q_i)$. To capture the demand stimulation, the demand modeling framework should allow the probability density function (Pdf) and the cumulative distribution function (Cdf) to vary with stocking quantity. Thus, referring to Rudi et al. (2001)；Balakrishnan et al. (2008)；Stavrulaki (2011), assume that the two stock-level dependent demands are independent, and thus for item i, $i=1,2$, define $f_i(Q_i,\cdot)$ as the pdf and $F_i(Q_i,x_i)=\int_{a_i(Q_i)}^{x_i} f_i(Q_i,\varepsilon)d\varepsilon$ as the Cdf for a given Q_i. Also, p_i is the unit retail price, w_i is the unit wholesale price and c_i is the unit production cost. Additionally, let $\alpha_i \in (0,1)$ denote a known substitution rate that customers buying item i will switch to the other if item i is out of stock.

2.1 The price-only contract

In the price-only contract, the retailer decides the two items' order quantities after the manufacturer's announcement of the two items' wholesale prices at the beginning of the selling period and then takes all responsibilities for leftover inventory at the end of the selling period. Accordingly, there are four scenarios in conjunction with the sales of the two items. Taking item 1 as an example:

Scenario 1: $x_1 > Q_1$

As demand x_1 is higher than the order quantity Q_1, it will result in lost sales of $x_1 - Q_1$ for item 1. Thus, Q_1 is item 1's sale whose expected value is given by

$$\int_{Q_1}^{b_1(Q_1)} \int_{a_2(Q_2)}^{b_2(Q_2)} Q_1 f_1(Q_1, x_1) f_2(Q_2, x_2) dx_2 dx_1 = \int_{Q_1}^{b_1(Q_1)} Q_1 f_1(Q_1, x_1) dx_1$$

Scenario 2: $x_1 < Q_1$ and $x_2 < Q_2$

The condition of $x_1 < Q_1$ implies an excess inventory of $Q_1 - x_1$ for item 1 that could be used as substitutes for item 2. However, $x_2 < Q_2$ infers that there exists no unsatisfied demand for item 2. Therefore, demand x_1 itself is the sale of item 1 whose expected value is obtained by

$$\int_{a_1(Q_1)}^{Q_1} \int_{a_2(Q_2)}^{Q_2} x_1 f_1(Q_1, x_1) f_2(Q_2, x_2) dx_2 dx_1$$

Scenario 3: $x_1 < Q_1$, $x_2 > Q_2$ and $x_1 + \alpha_2(x_2 - Q_2) < Q_1$ (or $x_2 < Q_2 + \dfrac{Q_1 - x_1}{\alpha_2}$)

The condition $x_1 < Q_1$ generates an excess inventory $Q_1 - x_1$ for item 1 that could substitute for item 2. The condition $x_2 > Q_2$ incurs an amount of unsatisfied demand of $x_2 - Q_2$ for item 2 that probably transfers to item 1. Therefore, according to the prior assumption, $\alpha_2(x_2 - Q_2)$ is the amount of unsatisfied demand that will switch to item 1. Thus, adding these additional sales, item 1's sale will increase to $x_1 + \alpha_2(x_2 - Q_2)$. If the $x_1 + \alpha_2(x_2 - Q_2)$ is less than the Q_1, it is item 1's sales and its expected value is calculated by

$$\int_{a_1(Q_1)}^{Q_1} \int_{Q_2}^{Q_2 + \frac{Q_1 - x_1}{\alpha_2}} (x_1 + \alpha_2(x_2 - Q_2)) f_1(Q_1, x_1) f_2(Q_2, x_2) dx_2 dx_1$$

Scenario 4: $x_1 < Q_1$, $x_2 > Q_2$ and $x_1 + \alpha_2(x_2 - Q_2) > Q_1$ (or $x_2 > Q_2 + \dfrac{Q_1 - x_1}{\alpha_2}$)

Similar to Scenario 3, the conditions $x_1 < Q_1$ and $x_2 > Q_2$ total item 1's sales up to $x_1 + \alpha_2(x_2 - Q_2)$, which is greater than Q_1, making Q_1 item 1's sales with the following expected value below.

$$\int_{a_1(Q_1)}^{Q_1} \int_{Q_2+\frac{Q_1-x_1}{\alpha_2}}^{b_2(Q_2)} Q_1 f_1(Q_1, x_1) f_2(Q_2, x_2) dx_2 dx_1$$

Combing the four scenarios together, item 1's total sales are obtained by

$$\begin{aligned}
E[z(Q_1)] = &\int_{Q_1}^{b_1(Q_1)} Q_1 f_1(Q_1, x_1) dx_1 + \int_{a_1(Q_1)}^{Q_1} \int_{a_2(Q_2)}^{Q_2} x_1 f_1(Q_1, x_1) f_2(Q_2, x_2) dx_2 dx_1 \\
&+ \int_{a_1(Q_1)}^{Q_1} \int_{Q_2}^{Q_2+\frac{Q_1-x_1}{\alpha_2}} (x_1 + \alpha_2(x_2 - Q_2)) f_1(Q_1, x_1) f_2(Q_2, x_2) dx_2 dx_1 \\
&+ \int_{a_1(Q_1)}^{Q_1} \int_{Q_2+\frac{Q_1-x_1}{\alpha_2}}^{b_2(Q_2)} Q_1 f_1(Q_1, x_1) f_2(Q_2, x_2) dx_2 dx_1 \\
= &\int_{Q_1}^{b_1(Q_1)} Q_1 f_1(Q_1, x_1) dx_1 + \int_{a_1(Q_1)}^{Q_1} \int_{a_2(Q_2)}^{Q_2+\frac{Q_1-x_1}{\alpha_2}} x_1 f_1(Q_1, x_1) f_2(Q_2, x_2) dx_2 dx_1 \\
&+ \int_{a_1(Q_1)}^{Q_1} \int_{Q_2}^{Q_2+\frac{Q_1-x_1}{\alpha_2}} \alpha_2(x_2 - Q_2) f_1(Q_1, x_1) f_2(Q_2, x_2) dx_2 dx_1 \\
&+ \int_{a_1(Q_1)}^{Q_1} \int_{Q_2+\frac{Q_1-x_1}{\alpha_2}}^{b_2(Q_2)} Q_1 f_1(Q_1, x_1) f_2(Q_2, x_2) dx_2 dx_1
\end{aligned}$$

Because $\displaystyle\int_{a_1(Q_1)}^{Q_1} \int_{a_2(Q_2)}^{Q_2+\frac{Q_1-x_1}{\alpha_2}} x_1 f_1(Q_1, x_1) f_2(Q_2, x_2) dx_2 dx_1$

$$\begin{aligned}
&= \int_{a_1(Q_1)}^{Q_1} \int_{a_2(Q_2)}^{b_2(Q_2)} x_1 f_1(Q_1, x_1) f_2(Q_2, x_2) dx_2 dx_1 - \int_{a_1(Q_1)}^{Q_1} \int_{Q_2+\frac{Q_1-x_1}{\alpha_2}}^{b_2(Q_2)} x_1 f_1(Q_1, x_1) f_2(Q_2, x_2) dx_2 dx_1 \\
&= \int_{a_1(Q_1)}^{Q_1} x_1 f_1(Q_1, x_1) dx_1 - \int_{a_1(Q_1)}^{Q_1} \int_{Q_2+\frac{Q_1-x_1}{\alpha_2}}^{b_2(Q_2)} x_1 f_1(Q_1, x_1) f_2(Q_2, x_2) dx_2 dx_1,
\end{aligned}$$

item 1's total sales can be expressed by

$$E[z(Q_1)] = \int_{a_1(Q_1)}^{Q_1} x_1 f_1(Q_1, x_1) dx_1 + \int_{Q_1}^{b_1(Q_1)} Q_1 f_1(Q_1, x_1) dx_1$$
$$+ \int_{a_1(Q_1)}^{Q_1} \int_{Q_2}^{Q_2 + \frac{Q_1 - x_1}{\alpha_2}} \alpha_2 (x_2 - Q_2) f_1(Q_1, x_1) f_2(Q_2, x_2) dx_2 dx_1$$
$$+ \int_{a_1(Q_1)}^{Q_1} \int_{Q_2 + \frac{Q_1 - x_1}{\alpha_2}}^{b_2(Q_2)} (Q_1 - x_1) f_1(Q_1, x_1) f_2(Q_2, x_2) dx_2 dx_1 \quad (1)$$

Obviously, the former two terms of $E[z(Q_1)]$ in Eq. (1) represent sales originated from item 1 itself with respect to $x_1 < Q_1$ and $x_1 > Q_1$, respectively. The latter two terms portray extra sales triggered by the effect of product substitution in response to $x_1 + \alpha_2(x_2 - Q_2) < Q_1$ and $x_1 + \alpha_2(x_2 - Q_2) > Q_1$, respectively. Similarly, item 2's sales can be constructed in the same way. Consequently, if let $Q = (Q_1, Q_2)$ for $i = 1, 2$, $j = 3 - i$, the retailer's total profit in the price-only contract is obtained by

$$\pi_r(Q) = \sum_{i=1}^{2} (p_i (\int_{a_i(Q_i)}^{Q_i} x_i f_i(Q_i, x_i) dx_i + \int_{Q_i}^{b_i(Q_i)} Q_i f_i(Q_i, x_i) dx_i$$
$$+ \int_{a_i(Q_i)}^{Q_i} \int_{Q_j}^{Q_j + \frac{Q_i - x_i}{\alpha_j}} \alpha_j (x_j - Q_j) f_i(Q_i, x_i) f_j(Q_j, x_j) dx_j dx_i$$
$$+ \int_{a_i(Q_i)}^{Q_i} \int_{Q_j + \frac{Q_i - x_i}{\alpha_j}}^{b_j(Q_j)} (Q_i - x_i) f_i(Q_i, x_i) f_j(Q_j, x_j) dx_j dx_i) - w_i Q_i) \quad (2)$$

After completing the $\pi_r(Q)$, the first-order necessary condition and the second-order sufficient condition maximizing the $\pi_r(Q)$ are respectively examined by the following two propositions whose proofs are similar to Stavrulaki (2011) and therefore are omitted for the sake of space.

Proposition 1: For $i = 1, 2$, $j = 3 - i$, the optimal order quantities in the price-only contract satisfy the following equation.

$$
\begin{aligned}
w_i = {} & p_i(1-F_i(Q_i,Q_i)-a_i(Q_i)f_i(Q_i,a_i(Q_i))a_i'(Q_i)+Q_i f_i(Q_i,b_i(Q_i))b_i'(Q_i) \\
& +\int_{a_i(Q_i)}^{Q_i} x_i \frac{\partial f_i(Q_i,x_i)}{\partial Q_i}dx_i+\int_{Q_i}^{b_i(Q_i)} Q_i \frac{\partial f_i(Q_i,x_i)}{\partial Q_i}dx_i \\
& -a_i'(Q_i)\int_{Q_j}^{Q_j+\frac{Q_i-a_i(Q_i)}{\alpha_j}} \alpha_j(x_j-Q_j)f_i(Q_i,a_i(Q_i))f_j(Q_j,x_j)dx_j \\
& +\int_{a_i(Q_i)}^{Q_i}\int_{Q_j}^{Q_j+\frac{Q_i-x_i}{\alpha_j}} \alpha_j(x_j-Q_j)\frac{\partial f_i(Q_i,x_i)}{\partial Q_i}f_j(Q_j,x_j)dx_j dx_i \\
& -a_i'(Q_i)\int_{Q_j+\frac{Q_i-a_i(Q_i)}{\alpha_j}}^{b_j(Q_j)} (Q_i-a_i(Q_i))f_i(Q_i,a_i(Q_i))f_j(Q_j,x_j)dx_j \\
& +\int_{a_i(Q_i)}^{Q_i}\int_{Q_j+\frac{Q_i-x_i}{\alpha_j}}^{b_j(Q_j)} (f_i(Q_i,x_i)+(Q_i-x_i)\frac{\partial f_i(Q_i,x_i)}{\partial Q_i})f_j(Q_j,x_j)dx_j dx_i) \\
& +p_j(\int_{a_j(Q_j)}^{Q_j}\int_{Q_i}^{Q_i+\frac{Q_j-x_j}{\alpha_i}} \alpha_i f_j(Q_j,x_j)((x_i-Q_i)\frac{\partial f_i(Q_i,x_i)}{\partial Q_i}-f_i(Q_i,x_i))dx_i dx_j \\
& +b_i'(Q_i)\int_{a_j(Q_j)}^{Q_j}(Q_j-x_j)f_j(Q_j,x_j)f_i(Q_i,b_i(Q_i))dx_j \\
& +\int_{a_j(Q_j)}^{Q_j}\int_{Q_i+\frac{Q_j-x_j}{\alpha_i}}^{b_i(Q_i)} (Q_j-x_j)f_j(Q_j,x_j)\frac{\partial f_i(Q_i,x_i)}{\partial Q_i}dx_i dx_j)
\end{aligned}
\tag{3}
$$

Proposition 2: For $i=1,2$, if item i’s stock-level dependent demand follows the uniform distribution $x_i \sim U[d_i\sqrt{Q_i}, s_i+d_i\sqrt{Q_i}]$, where d_i and s_i are positive constants such that $d_i<\sqrt{Q_i}$, then $\pi_r(Q)$ is jointly concave with respect to Q_i as long as $p_1>\alpha_2 p_2$ and $p_2>\alpha_1 p_1$.

As for the manufacturer’s total profit in the price-only contract, it is obtained by

$$\pi_m(w)=\sum_{i=1}^{2}(w_i-c_i)Q_i \text{ , where } w=(w_1,w_2) \tag{4}$$

Finally, the objective of the price-only contract is to determine the two wholesale prices that maximize the manufacturer’s profit subject to the retailer’s first-order necessary condition. That is,

$$\underset{w}{Max}\,\pi_m(w) \quad \text{s.t Eq. (3)} \tag{5}$$

The numerical solutions of Eq. (5) can be solved using Mathematica, and once the optimal wholesale prices, denoted by w_i^*, $i=1,2$, are identified, its corresponding optimal order quantity Q_i^*, optimal retailer profit π_r^* and optimal manufacturer profit π_m^* can be acquired by Eqs. (3), (2) and (4).

2.2 The centralized supply chain

Before proceeding to the return-policy contract, the centralized supply chain with a channel profit $\pi(Q)=\pi_r(Q)+\pi_m(Q)$ needs to be constructed first. Thus, from Eqs. (2) and (4), the following for $\pi(Q)$ is obtained.

$$\begin{aligned}\pi(Q)=\sum_{i=1}^{2}\Big(p_i\Big(&\int_{a_i(Q_i)}^{Q_i}x_i f_i(Q_i,x_i)dx_i+\int_{Q_i}^{b_i(Q_i)}Q_i f_i(Q_i,x_i)dx_i\\&+\int_{a_i(Q_i)}^{Q_i}\int_{Q_j}^{Q_j+\frac{Q_i-x_i}{\alpha_j}}\alpha_j(x_j-Q_j)f_i(Q_i,x_i)f_j(Q_j,x_j)dx_jdx_i\\&+\int_{a_i(Q_i)}^{Q_i}\int_{Q_j}^{Q_j+\frac{Q_i-x_i}{\alpha_j}}\alpha_j(x_j-Q_j)f_i(Q_i,x_i)f_j(Q_j,x_j)dx_jdx_i\\&+\int_{a_i(Q_i)}^{Q_i}\int_{Q_j+\frac{Q_i-x_i}{\alpha_j}}^{b_j(Q_j)}(Q_i-x_i)f_i(Q_i,x_i)f_j(Q_j,x_j)dx_jdx_i\Big)-c_iQ_i\Big)\end{aligned} \tag{6}$$

The goal of the centralized supply chain is to find the two optimal order quantities that maximize $\pi(Q)$. Therefore, if comparing the $\pi(Q)$ with the $\pi_r(Q)$ in Eq. (2) and according to Proposition 1, the following proposition regarding the two optimal order quantities in the centralized supply chain is concluded.

Proposition 3: For $i=1,2$, $j=3-i$, the optimal order quantities, denoted by Q_i^c, in the centralized supply chain satisfy the following equation.

$$\begin{aligned}c_i=p_i(1-F_i(Q_i^c,Q_i^c)&-a_i(Q_i^c)f_i(Q_i^c,a_i(Q_i^c))a_i'(Q_i^c)+Q_i^c f_i(Q_i^c,b_i(Q_i^c))b_i'(Q_i^c)\\&+\int_{a_i(Q_i^c)}^{Q_i^c}x_i\frac{\partial f_i(Q_i^c,x_i)}{\partial Q_i^c}dx_i+\int_{Q_i^c}^{b_i(Q_i^c)}Q_i^c\frac{\partial f_i(Q_i^c,x_i)}{\partial Q_i^c}dx_i\\&-a_i'(Q_i^c)\int_{Q_j^c}^{Q_j^c+\frac{Q_i^c-a_i(Q_i^c)}{\alpha_j}}\alpha_j(x_j-Q_j^c)f_i(Q_i^c,a_i(Q_i^c))f_j(Q_j^c,x_j)dx_j\\&+\int_{a_i(Q_i^c)}^{Q_i^c}\int_{Q_j^c}^{Q_j^c+\frac{Q_i^c-x_i}{\alpha_j}}\alpha_j(x_j-Q_j^c)\frac{\partial f_i(Q_i^c,x_i)}{\partial Q_i^c}f_j(Q_j^c,x_j)dx_jdx_i\\&-a_i'(Q_i^c)\int_{Q_j^c+\frac{Q_i^c-a_i(Q_i^c)}{\alpha_j}}^{b_j(Q_j^c)}(Q_i^c-a_i(Q_i^c))f_i(Q_i^c,a_i(Q_i^c))f_j(Q_j^c,x_j)dx_j\end{aligned}$$

$$
\begin{aligned}
&+\int_{a_i(Q_i^c)}^{Q_i^c}\int_{Q_j^c+\frac{Q_i^c-x_i}{\alpha_j}}^{b_j(Q_j^c)} \left(f_i(Q_i^c,x_i)+(Q_i^c-x_i)\frac{\partial f_i(Q_i^c,x_i)}{\partial Q_i^c}\right) f_j(Q_j^c,x_j)dx_j dx_i)\\
&+p_j(\int_{a_j(Q_j^c)}^{Q_j^c}\int_{Q_i^c}^{Q_i^c+\frac{Q_j^c-x_j}{\alpha_i}} \alpha_i f_j(Q_j^c,x_j)((x_i-Q_i^c)\frac{\partial f_i(Q_i^c,x_i)}{\partial Q_i^c}-f_i(Q_i^c,x_i))dx_i dx_j\\
&+b_i'(Q_i^c)\int_{a_j(Q_j^c)}^{Q_j^c}(Q_j^c-x_j)f_j(Q_j^c,x_j)f_i(Q_i^c,b_i(Q_i^c))dx_j\\
&+\int_{a_j(Q_j^c)}^{Q_j^c}\int_{Q_i^c+\frac{Q_j^c-x_j}{\alpha_i}}^{b_i(Q_i^c)} (Q_j^c-x_j)f_j(Q_j^c,x_j)\frac{\partial f_i(Q_i^c,x_i)}{\partial Q_i^c}dx_i dx_j)
\end{aligned}
\tag{7}
$$

Meanwhile, the optimal Q_i^c in Eq. (7) uniquely exists if the two items' stock-level dependent demands comply with Proposition 2. The optimal Q_i^c can be numerically determined with the aid of Mathematica's Newton method, and once Q_i^c is found, its corresponding optimal channel $\pi(Q^c)$ can be obtained by substituting $Q_i = Q_i^c$ in Eq. (6).

2.3 The return-policy contract

Having obtained the value of Q_i^c, $i=1,2$, the return-policy contract assumes that the manufacturer not only offers the two wholesale prices w_i^r cheaper than the optimal w_i^* in the price-only contract but also accepts all unsold products with a unit buyback price of v_i at the end of the selling period in order to entice the retailer's orders up to Q_i^c. Hence, according to Scenarios 2 and 3 mentioned before, item 1's total returns are given by

$$
\begin{aligned}
&\int_{a_1(Q_1^c)}^{Q_1^c}\int_{a_2(Q_2^c)}^{Q_2^c}(Q_1^c-x_1)f_1(Q_1^c,x_1)f_2(Q_2^c,x_2)dx_2dx_1\\
&\quad+\int_{a_1(Q_1^c)}^{Q_1^c}\int_{Q_2^c}^{Q_2^c+\frac{Q_1^c-x_1}{\alpha_2}}(Q_1^c-(x_1+\alpha_2(x_2-Q_2^c)))f_1(Q_1^c,x_1)f_2(Q_2^c,x_2)dx_2dx_1\\
&=\int_{a_1(Q_1^c)}^{Q_1^c}\int_{a_2(Q_2^c)}^{Q_2^c+\frac{Q_1^c-x_1}{\alpha_2}}(Q_1^c-x_1)f_1(Q_1^c,x_1)f_2(Q_2^c,x_2)dx_2dx_1\\
&\quad-\int_{a_1(Q_1^c)}^{Q_1^c}\int_{Q_2^c}^{Q_2^c+\frac{Q_1^c-x_1}{\alpha_2}}\alpha_2(x_2-Q_2^c)f_1(Q_1^c,x_1)f_2(Q_2^c,x_2)dx_2dx_1
\end{aligned}
$$

Therefore, if incorporating these returns into Eqs. (2) and (4), the retailer's profit, denoted by π_r^r, and the manufacturer's profits, denoted by π_m^r, in the return-policy contract are given as follows. For $i=1,2$, $j=3-i$, $w^r=(w_1^r,w_2^r)$,

$$\pi_r^r = \sum_{i=1}^{2}\Big(p_i\Big(\int_{a_i(Q_i^c)}^{Q_i^c} x_i f_i(Q_i^c, x_i)dx_i + \int_{Q_i^c}^{b_i(Q_i^c)} Q_i^c f_i(Q_i^c, x_i)dx_i$$
$$+\int_{a_i(Q_i^c)}^{Q_i^c}\int_{Q_j^c}^{Q_j^c+\frac{Q_i^c-x_i}{\alpha_j}} \alpha_j(x_j - Q_j^c) f_i(Q_i^c, x_i) f_j(Q_j^c, x_j)dx_j dx_i$$
$$+\int_{a_i(Q_i^c)}^{Q_i^c}\int_{Q_j^c+\frac{Q_i^c-x_i}{\alpha_j}}^{b_j(Q_j^c)} (Q_i^c - x_i) f_i(Q_i^c, x_i) f_j(Q_j^c, x_j)dx_j dx_i\Big) \tag{8}$$
$$+v_i\Big(\int_{a_i(Q_i^c)}^{Q_i^c}\int_{a_j(Q_j^c)}^{Q_j^c+\frac{Q_i^c-x_{1i}}{\alpha_j}} (Q_i^c - x_i) f_i(Q_i^c, x_i) f_j(Q_j^c, x_j)dx_j dx_i$$
$$-\int_{a_i(Q_i^c)}^{Q_i^c}\int_{Q_j^c}^{Q_j^c+\frac{Q_i^c-x_i}{\alpha_j}} \alpha_j(x_j - Q_j^c) f_i(Q_i^c, x_i) f_j(Q_j^c, x_j)dx_j dx_i\Big) - w_i^r Q_i^c\Big)$$

$$\pi_m^r = \sum_{i=1}^{2}\Big((w_i^r - c_i)Q_i^c$$
$$-v_i\Big(\int_{a_i(Q_i^c)}^{Q_i^c}\int_{a_j(Q_j^c)}^{Q_j^c+\frac{Q_i^c-x_{1i}}{\alpha_j}} (Q_i^c - x_i) f_i(Q_i^c, x_i) f_j(Q_j^c, x_j)dx_j dx_i \tag{9}$$
$$-\int_{a_i(Q_i^c)}^{Q_i^c}\int_{Q_j^c}^{Q_j^c+\frac{Q_i^c-x_i}{\alpha_j}} \alpha_j(x_j - Q_j^c) f_i(Q_i^c, x_i) f_j(Q_j^c, x_j)dx_j dx_i\Big)\Big)$$

From Eqs. (8), (9) and (6), the coordination of the contract is clearly identified due to $\pi_r^r + \pi_m^r = \pi(Q^c)$.

3. Analysis

For simplicity, $i = 1,2$, $j = 3 - i$, and let

$$P = \sum_{i=1}^{2}\Big(p_i\Big(\int_{a_i(Q_i^c)}^{Q_i^c} x_i f_i(Q_i^c, x_i)dx_i + \int_{Q_i^c}^{b_i(Q_i^c)} Q_i^c f_i(Q_i^c, x_i)dx_i$$
$$+\int_{a_i(Q_i^c)}^{Q_i^c}\int_{Q_j^c}^{Q_j^c+\frac{Q_i^c-x_i}{\alpha_j}} \alpha_j(x_j - Q_j^c) f_i(Q_i^c, x_i) f_j(Q_j^c, x_j)dx_j dx_i$$
$$+\int_{a_i(Q_i^c)}^{Q_i^c}\int_{Q_j^c+\frac{Q_i^c-x_i}{\alpha_j}}^{b_j(Q_j^c)} (Q_i^c - x_i) f_i(Q_i^c, x_i) f_j(Q_j^c, x_j)dx_j dx_i\Big)\Big)$$

$$R_i = \int_{a_i(Q_i^c)}^{Q_i^c} \int_{a_j(Q_j^c)}^{Q_j^c + \frac{Q_i^c - x_{1i}}{\alpha_j}} (Q_i^c - x_i) f_i(Q_i^c, x_i) f_j(Q_j^c, x_j) dx_j dx_i$$

$$= \int_{a_i(Q_i^c)}^{Q_i^c} \int_{a_j(Q_j^c)}^{Q_j^c + \frac{Q_i^c - x_{1i}}{\alpha_j}} (Q_i^c - x_i) f_i(Q_i^c, x_i) f_j(Q_j^c, x_j) dx_j dx_i$$

$$- \int_{a_i(Q_i^c)}^{Q_i^c} \int_{Q_j^c}^{Q_j^c + \frac{Q_i^c - x_i}{\alpha_j}} \alpha_j (x_j - Q_j^c) f_i(Q_i^c, x_i) f_j(Q_j^c, x_j) dx_j dx_i$$

$$V = \sum_{i=1}^{2} v_i R_i \quad \text{and} \quad C = \sum_{i=1}^{2} c_i Q_i^c$$

Note that the values of P, R_i and C are already determined because the value Q_i^c has been previously obtained, and V will also be determined if the buyback price $v = (v_1, v_2)$ is set. This suggests that π_r^r in Eq. (8) and π_m^r in Eq. (9) will be linear in w_i^r if $v = (v_1, v_2)$ is set, and can thus be simplified as follows.

$$\pi_r^r = -\sum_{i=1}^{2} Q_i^c w_i^r + P + V \tag{10}$$

$$\pi_m^r = \sum_{i=1}^{2} Q_i^c w_i^r - V - C \tag{11}$$

Thus, $\pi_r^r > \pi_r^*$ and $\pi_m^r > \pi_m^*$ are needed to ensure both members' better profits, and this leads to the following constraint for w_i^r, along with Fig 1 where

$$\overleftrightarrow{AB} : \sum_{i=1}^{2} Q_i^c w_i^r = V + C + \pi_m^* \quad \text{and} \quad \overleftrightarrow{CD} : \sum_{i=1}^{2} Q_i^c w_i^r = P + V - \pi_r^* .$$

$$\begin{cases} (w_1^r, w_2^r) < (w_1^*, w_2^*) \\ V + C + \pi_m^* < \sum_{i=1}^{2} Q_i^c w_i^r < P + V - \pi_r^* \end{cases} \tag{12}$$

Furthermore, from Fig 1, the following proposition with respect to the two negotiated wholesale prices is made.

Proposition 4: Compared to the price-only contract, if (w_1^r, w_2^r) is negotiated such that

(1)it lies in region I of Fig 1, then both members will receive more profits from the game;

(2)it is in region II of Fig 1, then only the manufacturer receives more profits from the game;

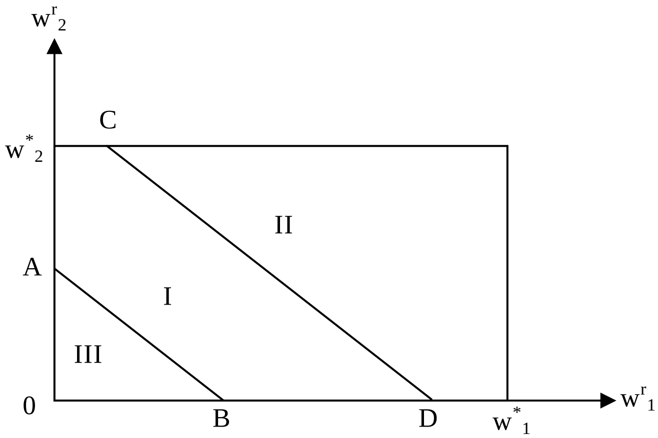

Fig 1 The range of (w_1^r, w_2^r)

(3)it locates in region III of Fig 1, then only the retailer earns more profits from the game.

Clearly, region I of Fig 1 is the range for (w_1^r, w_2^r) to create a win-win game. More specifically, the location of (w_1^r, w_2^r) determines which member will benefit more from the game. If the manufacturer dominates the game, he will negotiate (w_1^r, w_2^r) close to $\overline{CD}$ for a higher profit. Contrarily, if the retailer dominates the game, he will negotiate (w_1^r, w_2^r) close to $\overline{AB}$ for better profits.

Additionally, the two parallel lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ will simultaneously move towards the right-hand side of the region $[0, w_1^*] \times [0, w_2^*]$ as (v_1, v_2) increases. Once (v_1, v_2) is so large that $\overleftrightarrow{AB}$ is passing through the point (w_1^*, w_2^*) (see Fig 2), then based on Proposition 4, the game would no longer be profitable to the manufacturer. At this moment, $\overleftrightarrow{AB}$ satisfies $\sum_{i=1}^{2} Q_i^c w_i^* = V + C + \pi_m^*$, and thus $\sum_{i=1}^{2} R_i v_i = \sum_{i=1}^{2} Q_i^c w_i^* - C - \pi_m^*$ as $V = \sum_{i=1}^{2} v_i R_i$. Accordingly, if (v_1, v_2) is set such that $\sum_{i=1}^{2} R_i v_i > \sum_{i=1}^{2} Q_i^c w_i^* - C - \pi_m^*$, the manufacturer would never receive better benefits. Therefore, let $\overleftrightarrow{EF} : \sum_{i=1}^{2} R_i v_i = \sum_{i=1}^{2} Q_i^c w_i^* - C - \pi_m^*$, the following proposition in association with Fig 3 is then drawn to respond to the setting of (v_1, v_2) .

Proposition 5: With respect to (v_1, v_2) ,

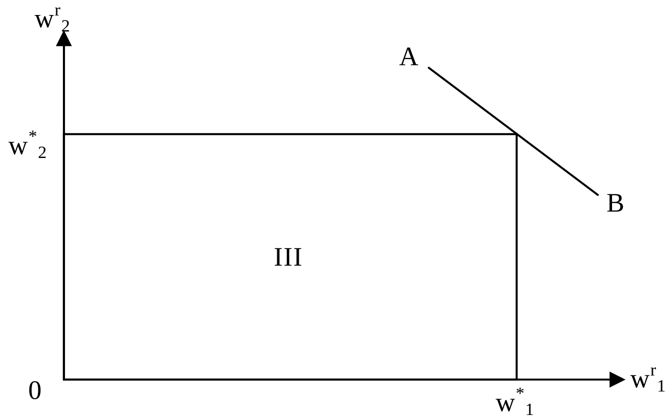

Fig 2 The non-profitable game for manufacturer

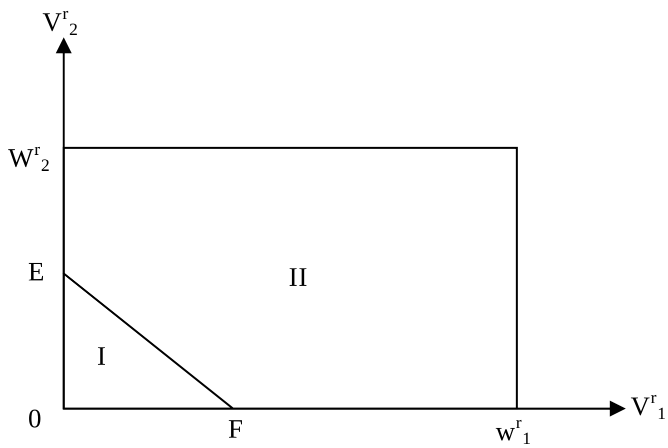

Fig 3 The range of (v_1^r, v_2^r)

(1) in practice, a manufacturer usually sets a buyback price lower than its wholesale price to avoid arbitrage transaction, which limits $(v_1, v_2) \in (0, w_1^r) \times (0, w_2^r)$, as shown in Fig 3;

(2) if (v_1, v_2) is set in region II of Fig 3, it is impossible for the manufacturer to obtain more profits compared to the price-only contract;

(3) only region I in Fig 3 can provide the manufacturer a better profit; summarily, (v_1, v_2) should be in line with the following inequalities.

$$\begin{cases} (v_1, v_2) < (w_1^r, w_2^r) \\ \sum_{i=1}^{2} R_i v_i < \sum_{i=1}^{2} Q_i^c w_i^* - C - \pi_m^* \end{cases} \tag{13}$$

4. Examples

The parameter values are assumed that, for $i = 1,2$, the stock-level dependent demand $x_i \sim U[d_i\sqrt{Q_i}, s_i + d_i\sqrt{Q_i}]$ with s_i =200, d_i =1 and the retail price p_i =20. Tables 1 and 2 compare the retailer's, the manufacturer's and the chain's profits in the return-policy contract with those in the price-only contract in response to a rising production cost and a various substitution rate, respectively. Table 3 then discusses how (w_1^r, w_2^r) will impact the profit performances when concurrently facing a rising production cost as well as an increasing substitution rate.

Recall that in the course of the examples, optimal values in the price-only contract are needed to be solved beforehand in the order of w_i^* by Eq. (5), Q_i^* by Eq. (3), π_r^* by Eq. (2) and π_m^* by Eq. (4), and let π^*=π_r^*+π_m^*. The next are the centralized supply chain's optimal order Q_i^c by Eq. (7) and optimal profit π^c by Eq. (6). Then, the manufacturer announces v_i according to Eq. (13) and negotiates w_i^r with the retailer based on Eq. (12). Once v_i and w_i^r are set, the retailer's π_r^r and the manufacturer's profits π_m^r in the return-policy contract are obtained by Eqs. (10) and (11).

For convenience, assume (v_1, v_2) =(5,5) and (w_1^r, w_2^r) =(12,12) for all examples in Tables 1 and 2, both of which have been confirmed in accordance with Eqs. (13) and (12) except for the case c_2 =6 in Table 1, where π_m^r =1578.42< π_m^* =1584.04 because of the unsatisfied Eq. (12).

In Table 1, fix c_1 =4, enlarge c_2 from c_2 =2 to c_2 =6 and keep other parameters unchanged to determine how the increasing c_2 will influence the game. First, it shows that Q_1^c increases from Q_1^c =101.68 to Q_1^c =168.31, whereas Q_2^c decreases from Q_2^c =174.07 to Q_2^c =91.34, which suggests that the chain should cut sales with expensive production costs and then offset the lost sales with cheaper production costs. Still, it undermines the chain's profit π^c from π^c =3477.34 to π^c =2938.04. Compared with π^*

Table 1 Results of $\alpha_1=\alpha_2$=0.7, c_1=4, (v_1,v_2)=(5,5), (w_1^r,w_2^r)=(12,12), $\rho_r+\rho_m=\rho$

c_2	$(Q_1^*,Q_2^*)/(Q_1^c,Q_2^c)$	π^*/π^c	ρ	(w_1^*,w_2^*)	$(\pi_r^*,\pi_m^*)/(\pi_r^r,\pi_m^r)$	(ρ_r,ρ_m)
2	(59.66,101.30)	2174.05		(14.94,14.28)	(277.33,1896.72)	
	(101.68,174.07)	3477.34	59.9%		(1302.51,2174.83)	(47.2%,12.8%)
3	(69.00,90.15)	2069.97		(14.93,14.61)	(269.01,1800.96)	
	(117.95,154.70)	3347.67	61.7%		(1371.44,1976.23)	(53.3%,8.5%)
4	(78.51,78.51)	1946.90		(14.93,14.93)	(230.31,1716.59)	
	(134.49,134.49)	3217.53	65.3%		(1408.16,1809.36)	(60.5%,4.8%)
5	(88.16,66.36)	1801.46		(14.93,15.26)	(157.35,1644.11)	
	(151.29,113.39)	3083.19	71.1%		(1407.07,1676.11)	(69.4%,1.8%)
6	(97.93,53.68)	1630.32		(14.92,15.58)	(46.28,1584.04)	
	(168.31,91.34)	2938.04	80.3%		(1359.63,1578.42)	(80.5%,-0.3%)

Table 2 Results of α_1=0.7, $c_1=c_2=4$, (v_1,v_2)=(5,5), (w_1^r,w_2^r)=(12,12), $\rho_r+\rho_m=\rho$

α_2	$(Q_1^*,Q_2^*)/(Q_1^c,Q_2^c)$	π^*/π^c	ρ	(w_1^*,w_2^*)	$(\pi_r^*,\pi_m^*)/(\pi_r^r,\pi_m^r)$	(ρ_r,ρ_m)
0.5	(62.83,90.91)	1944.32		(14.94,14.93)	(263.23,1681.09)	
	(107.34,156.01)	3268.96	68.1%		(1497.36,1771.60)	(63.5%,4.7%)
0.6	(70.31,85.06)	1955.58		(14.94,14.93)	(256.89,1698.69)	
	(120.27,145.87)	3258.84	66.6%		(1468.53,1790.31)	(62.0%,4.7%)
0.7	(78.51,78.51)	1946.90		(14.93,14.93)	(230.31,1716.59)	
	(134.49,134.49)	3217.53	65.3%		(1408.16,1809.36)	(60.5%,4.8%)
0.8	(88.15,70.66)	1908.46		(14.92,14.93)	(172.72,1735.74)	
	(151.27,120.79)	3128.13	63.9%		(1298.19,1829.94)	(59.0%,4.9%)
0.9	(100.68,60.18)	1811.87		(14.92,14.94)	(54.15,1757.72)	
	(173.25,102.37)	2946.84	62.6%		(1093.13,1853.71)	(57.3%,5.3%)

Table 3 Results of α_1=0.7, c_1=4, (v_1,v_2)=(5,5)

	For α_2=0.7, c_2=6	For α_2=0.5, c_2=4	For α_2=0.5, c_2=6
(w_1^r,w_2^r)	(π_r^r,π_m^r)	(π_r^r,π_m^r)	(π_r^r,π_m^r)
(12,13)	(1268.29,1669.76)	(1341.35,1927.61)	(1553.55,1598.14)
(12,13.5)	(1222.62,1715.43)	(1263.35,2005.62)	(1491.44,1660.25)
(12,14)	(1176.95,1761.10)	(1185.34,2083.62)	(1429.33,1722.36)
(12,14.5)	(1131.28,1806.77)	(1107.34,2161.63)	(1367.22,1784.47)
(13,12)	(1191.32,1746.73)	(1390.02,1878.94)	(1548.95,1602.74)
(13.5,12)	(1107.16,1830.88)	(1336.35,1932.61)	(1484.54,1667.15)
(14,12)	(1023.01,1915.04)	(1282.68,1986.28)	(1420.13,1731.56)
(14.5,12)	(938.85,1999.19)	(1229.01,2039.95)	(1355.72,1795.97)

in Table 1, however, it is shown that the higher the production cost c_2 is, the greater the chain profit increment $\rho = \frac{\pi^c - \pi^*}{\pi^*}$ is as it increases from ρ =59.9% at c_2 =2 to ρ =80.3% at c_2 =6.

Second, the rising c_2 slashes the manufacturer's profit π_m^r from π_m^r =2174.38 to π_m^r =1578.42 and profit increment $\rho_m = \frac{\pi_m^r - \pi_m^*}{\pi^*}$ from ρ_m =12.8% to ρ_m =-0.3%. The retailer's profit π_r^r and profit increment $\rho_r = \frac{\pi_r^r - \pi_r^*}{\pi^*}$, however, increase from π_r^r =1302.51 with ρ_r =47.2% at c_2 =2 to a peak π_r^r =1408.16 with ρ_r =60.5% at c_2 =4 and then to π_r^r =1359.63 with ρ_r =80.5% at c_2 =6. All told, the manufacturer alone assumes all risks and the retailer shares most benefits from the rising production cost.

Table 2 suggests that how a various substitution rate will impact the chain. Therefore, α_1 =0.7 is fixed, α_2 is increased from α_2 =0.5 to α_2 =0.9 and other parameters remain unchanged. First, it is conceivable that a higher substitution rate of an item allows a reduced order quantity of that item and increases sales of its substitute, which explains why Q_2^c decreases from Q_2^c =156.01 to Q_2^c =102.37, while Q_1^c increases from Q_1^c =107.34 to Q_1^c =173.25 as α_2 increases from α_2 =0.5 to α_2 =0.9. Overall, the chain's total sales $Q^c = Q_1^c + Q_2^c$ increase from $Q^c = 263.25$ to $Q^c = 275.62$, and this result urges that a chain should maintain a larger inventory when managing substitutable items. The larger inventory Q^c, however, does not benefit the chain's profit π^c which decreases from π^c =3268.96 to π^c =2946.84.

Second, it is understandable that the more total sales $Q^c = Q_1^c + Q_2^c$, the more the manufacturer's revenue $w_1^r Q_1^c + w_2^r Q_2^c$ as $w_1^r = w_2^r$ is assumed, explaining why the manufacturer's π_m^r increases from π_m^r =1771.60 to π_m^r =1853.71 in the wake of the increasing α_2 from α_2 =0.5 to α_2 =0.9. Conversely, the increasing α_2 damages the retailer's π_r^r from π_r^r =1497.36 to π_r^r =1093.13, which is attributed to that a higher α_2 yields more item 2 sales switching to item 1 with a revenue margin of $\alpha_2 p_1$ that is lower than the original p_2. And the decrease in π_r^r is more than the increase in π_m^r, which explains why the chain's profit π^c decreases when α_2 increases. Table 2 also reveals that a higher α_2 increases the manufacturer's profit π_m^r with a profit increment from ρ_m =4.7% to ρ_m =5.3%. A higher α_2 reduces the retailer's profit π_r^r, though his profit increment still remains at the relative high level of ρ_r =57.3% even though α_2 reaches 0.9.

Table 3 attempts to understand how (w_1^r, w_2^r) will affect the game. Thus, fix α_1=0.7, c_1=4, set (v_1, v_2)=(5,5) and interchange the values of (w_1^r, w_2^r) in Table 3 such that the following are gained.

In response to α_2=0.7(=α_1), c_2=6(>c_1), Table 3 shows a contra-intuitive outcome that a cheaper wholesale price w_2^r responding to the higher production cost c_2 generates greater manufacturer profit, e.g., π_m^r=1746.73 at (w_1^r, w_2^r)=(13,12) is better than π_m^r=1669.76 at (w_1^r, w_2^r)=(12,13), which is credited to Q_2^c=91.34<Q_1^c=168.31 resulting from $c_2 > c_1$, as explained in Table 1.

In response to α_2=0.5(<α_1), c_2=4(=c_1), Table 3 suggests that a pricy w_2^r, by comparison, contributes greater profit to the manufacturer, e.g., π_m^r=1927.61 at (w_1^r, w_2^r)=(12,13) is higher than π_m^r=1878.94 at (w_1^r, w_2^r)=(13,12), which is supported by Q_2^c=156.01>Q_1^c=107.34, the result of $\alpha_2 < \alpha_1$, as explained in Table 2.

Thus, from the manufacturer's aspect, $c_2 > c_1$ implies $Q_2^c < Q_1^c$ and thus $w_2^r < w_1^r$, and $\alpha_2 < \alpha_1$ implies $Q_2^c > Q_1^c$ and thus $w_2^r > w_1^r$. As regards α_2=0.5(<α_1), c_2=6(>c_1), Table 3 indicates the effect of rising production costs overwhelms the effect of product substitution by showing that Q_2^c=124.22<Q_1^c=128.82, which prompts the manufacturer to prioritize a pricy w_1^r for a better profit, e.g., π_m^r=1602.74 at (w_1^r, w_2^r)=(13,12) is better than π_m^r=1598.14 at (w_1^r, w_2^r)=(12,13).

5. Conclusion

This study examined inventory management of two substitutable newsvendor-type items from the perspective of a one-manufacturer, one-retailer decentralized supply chain, taking the effects of product substitution and inventory demand stimulation into consideration. To increase the retailer's orders to the level of a centralized supply chain, the manufacturer offers the two cheaper wholesale prices by comparison with the price-only contract and accepts all unsold products at the end of the selling period.

From channel profit perspective, the effects of product substitution and of inventory demand stimulation are basically antagonistic. A high substitution rate allows a smaller inventory to avoid the risk of over-stock, but it decreases the effect of inventory demand stimulation and increases the probability of switching to a meager substitute's profit margin. However, while a large inventory can stimulate sales, it increases the risk of over-stock and decreases a potential of switching to a more profitable substitute. Taking Table 2 as an example, because $c_1 = c_2$ and $p_1 = p_2$, $\alpha_2 p_1 < p_2$ suggests that the substitute item 1's profit margin $\alpha_2 p_1 - c_1$ is less than the original item 2's $p_2 - c_2$, thereby impairing the chain's profit.

Additionally, during the course of this study, the following are resolved. (1)The return-policy contract with cheaper wholesale prices and buyback commitment was confirmed to coordinate the chain. (2)The contract is Pareto-efficient if the negotiated wholesale prices conform to Eq. (12). (3)This paper provided the manufacturer with Eq. (13) to set the two buyback prices. (4)The contract was proven indispensable for a better channel profit, especially for high production costs. (5)The manufacturer alone assumes all risks during the game. (6)The chain should build up a large stockpile of inventory when dealing with substitutable items, especially for items with high substitution rates.

A related study to this paper is a two-manufacturer, one-retailer supply chain setting where the two manufacturers supply an item to the retailer who then makes his decisions according to not only the two manufacturers' offers but also the negotiations among them. The negotiations could be bilateral between the retailer and each of the two manufacturers or trilateral among them. The game could also involve the two manufacturers' supply certainties and / or uncertainties, such as Serel (2008) who maximized a retailer expected profit by allocating order between a certain supplier and an uncertain supplier. Compared to a single-item setting, a two-substitutable-item one is much more complicated and challenging but worthy of a further exploring. For other future studies, the proposed models can be modified by incorporating a stock-level, price dependent demand because retail prices profoundly influence consumers' purchasing behavior. The proposed setting can also be extended to a one-manufacturer, one-retailer or a one-manufacture, multiple-retailer one with multiple substitutable items.

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